Virtual and Hands-On Experiments in Statics: Balance Properties of Asymmetrical Bodies

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Combining teaching physics and mathematics at the college level is a big educational challenge despite well-understood historical interaction between both fields. We believe that a successful implementation of an integrated physics/mathematics course can be accomplished by allowing students to first analyze physical phenomena and next by challenging them to develop and learn the underlying mathematics. We illustrate the proposed educational approach by using the study of balance properties of various objects; in particular by posing a question on how hollowness affects the balance of asymmetrical bodies. To aid students in formulating by themselves a proper mathematical description, we have designed an interactive computer program that determines the coordinates of the mass centers for a variety of solid and hollow bodies. Consequently, students not only gain an understanding of how mathematics is used to model physical laws but also how mathematical formulas are implemented in computer modeling of physical phenomena.

Introduction

Since ancient times, scientific observations have stimulated the development of mathematics and *vice versa*—mathematics discoveries have lead to new scientific theories. Indeed, astronomical and physical observations made by Babylonians and Greeks enhanced the development of Geometry and Number Theory that in turn enabled them to create models of our Universe. Attempts to refine these models fostered further mathematical discoveries. Similarly, the invention of Calculus, which was triggered mainly by physics, led to an incredible progress in sciences and mathematics. Finally, in the 20th century, the interaction between Functional Analysis, Group Theory, Measure Theory, etc., on one side and Relativity Theories, Quantum Theory, and Elementary Particles Theories on the other accelerated the advancement of modern mathematics and sciences.

Many educators try to emulate this historical interactive 'success story' while teaching physics and mathematics at the college level. However, teaching an integrated physics/mathematics course without students having prior solid basic mathematical preparation is not always possible. Consequently, it has been very difficult to implement such approaches outside of the top universities. Indeed, the Feynman Lectures on Physics developed in the 1960s at the California Institute of Technology [1] assume the mastery level in Calculus, Differential Equations, etc., that is not usually achieved by students at an average college (especially not during their first years of studies). Similarly, the extensive mathematics program developed in the 1960s (and still used) for physics majors at the Warsaw University by Prof. K. Maurin [2, 3] (enabling physicists to teach complicated physics topics during the freshman year and to introduce advanced physics theories as early as the sophomore year) has never been successfully disseminated in Poland beyond the Warsaw University. In 1970-1980s, a co-author of the paper, Prof. I. Szczyrba, during his tenure at that university implemented a simplified version of the program for prospective physics teachers. Even that version appeared to be too difficult for implementation at other colleges in Poland.

In 1980s, Prof. D. Goodstein, a distinguished educator at the California Institute of Technology, attempted to develop a calculus course that was rooted in history and would become the basis of physics courses for nearly everyone. According to him, the experiment failed due to inadequate high school preparation of students in trigonometry [4]. In the last twenty years all three co-authors of the paper attempted to emulate in various colleges in Europe and the U.S. their extensive integrated mathematics/physics education obtained in elite Soviet era schools—and all three of them had similar negative experiences as Prof. Goodstein had. The educational reality in any country seems to be that instructor should always be ready to face gaps (if not abysses) in the students' mathematical knowledge essential for understanding any lectured mathematics or science course.

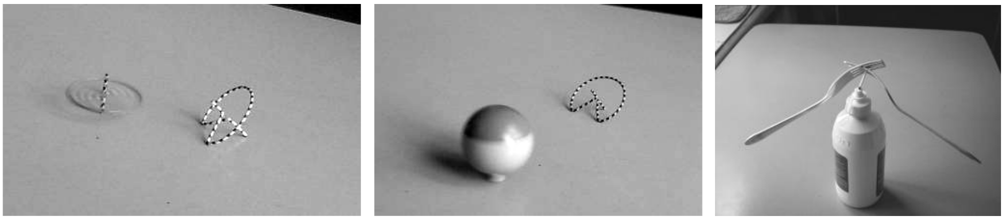
We follow Prof. Goodstein's challenge to 'devise ways to teach physics that will make the subject so vital and appealing that it will be unthinkable for any educated person in the twenty-first century not to have mastered its elements' [4]. The premise of our history-motivated approach is to allow an average student to first observe and analyze some simple but interesting physical phenomena and next to challenge him or her to develop and learn the underlying mathematical description.

Mathematics behind some tricky balances and spectacular tops

Many physics toys and tricks successfully used to enliven a physics course are also a great resource for students' motivation to learn mathematics. For instance, a nice sample topic is studying the consequences of the positioning of the center of mass of a specific body at rest or in motion. A challenging problem for freshman students within this topic is to make a stable-rotating asymmetrical top from a paper clip — a Sakai top [5, 6]. Some of the clip's wire is used to create the rotational axis whereas the rest is shaped into a circular arc closed by two radial spokes (Fig. 1*a*). Importantly, prior to application of their hands-on skills, students must calculate an angle α between the spokes that leads to the positioning of the top's center of mass at its rotational axis. The solution is rewardingly simple: $\tan(\alpha/2) = ½$ as shown in Fig. 1*b* right.

Significantly more complicated mathematics is needed to describe dynamics of the famous tippe-top that turns upside-down when it is rotated on a flat surface (Fig. 1b left) [7, 8]. But again, it is the position of the center of mass that is crucial for the toy to operate. Thus, in the typical case of a spherical shape of the toy's base, the center of the sphere should be above the toy's center of mass.

An attempt to balance two forks using an inserted match (a toothpick or a coin) on a sharp edge or on a tiny spot (Fig. 1*c*) is another amusing and spectacular trick rooted in the positioning of the system's center of mass [9]. Students first solve this problem experimentally by adjusting the angle between the forks to keep the center of mass below the suspension point. More mathematically advanced students can be asked to estimate the position of the center of mass and to calculate the period of oscillations of the obtained physical pendulum—to be compared with the experimentally measured value.



a *b* *c*

Figure 1: Physics toys and tricks involving the positioning of centers of mass. Sakai paper-clip top (*a, b* right); tippe-top (*b* left); balancing of coupled forks with a match (*c*).

Amazingly balanced natural and constructed objects

Geologists, architects, engineers, and builders are often required to decide whether a given natural or artificial body is gravitationally stable. In particular, the fate of the leaning Tower of Pisa has intrigued humankind for centuries and in the last several decades the efforts to rescue the Tower have become a front-page story. There are other similar amazingly-balanced constructions, monuments, and natural objects that—while not quite as famous—have been catching the observer's eye all around the world as well (Fig. 2).

It is very instructional and interesting for students to determine the positions of the centers of mass of these objects using geometrical terms and next, following the fundamental law of Statics, to decide under which circumstances a particular object can tumble down. One can assume in such considerations that natural bodies have a homogeneous distribution of mass, i.e., a constant density. However, such an assumption is usually not valid for the artificially constructed objects. Indeed, in contrast to the solid boulder depicted in Fig. 2*a,* the metal monument in Fig. 2c is hollow.

a *b* *c*

Figure 2: Are these bodies safe in balance? A spectacular balanced rock in the Garden of the Gods natural park, Colorado Springs, U.S.A (*a*). Balancing stony towers on the shore of Lake Ontario, Canada (*b*). A modern sculpture in a city park, Greeley, Colorado, U.S.A (*c*).

Balance properties of hollow bodies in virtual and real experiments

Our project allows students to study how the hollowness affects the balance properties of asymmetrical bodies. The basic mathematical questions to be investigated by students are:

(i) how far apart are the centers of mass of a solid object and of the corresponding hollow shell?

(ii) does the projection of the center of mass remain within the base of the object when the mass is gradually removed from inside of a gravitationally stable solid object?

In their investigation of balance properties of asymmetrical solid and hollow bodies students can focus first on flat triangular figures. It is known that the center of mass of a solid triangle coincides with the center of mass of the corresponding 'hollow' triangle (the triangle's boundary) only if the triangle is equilateral. Furthermore, in an arbitrary triangle both mass centers may become separated in an essential way [10]. To help students in their investigation, we have designed an interactive computer research program (written in Microsoft Visual Basic 6.0) that determines the coordinates of the mass centers for a variety of solid and hollow bodies resting on a horizontal or inclined plane. Gradually changing the shape and the hollowness of objects studied, students will observe whether the projection of the body's center of mass remains within the body's base.

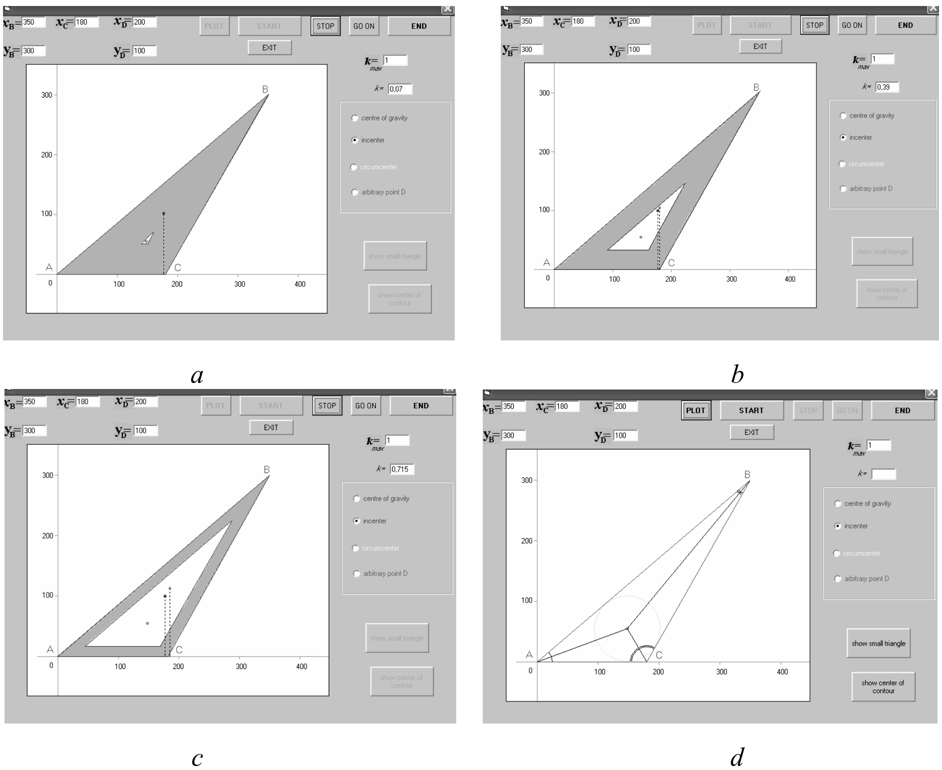


Figure 3: Cutting a similar triangle out of an arbitrary triangle. Dotted vertical lines depict the projections of the mass centers of the solid and the partially hollow triangle onto the base *AC*.

In particular, cutting off a very small triangle inside of a given solid triangle in such a way that the sides of both triangles remain parallel practically does not change the position of the center of mass (Fig 3a). Increasing the size of the cut-off triangle allows students to see that the center of mass of the remaining figure moves from its initial position (Fig. 3b and 3c). When the projection of the center of mass leaves the base AC, the virtual body tumbles down. The situation where the distances between each of the three pairs of the triangles' parallel sides are equal, as is the case in screenshots depicted in Fig. 3, is particularly interesting from the mathematical point of view. In that case, the straight lines connecting the corresponding pairs of vertices in both similar triangles intersect at the center of the circle inscribed into the initial triangle (Fig. 3d).

Computer simulations of figures studied are followed by hands-on experiments with their cardboard replicas built by students. The three-dimensional triangular figures allow emulating the balance properties of flat triangles as well (Fig 4).

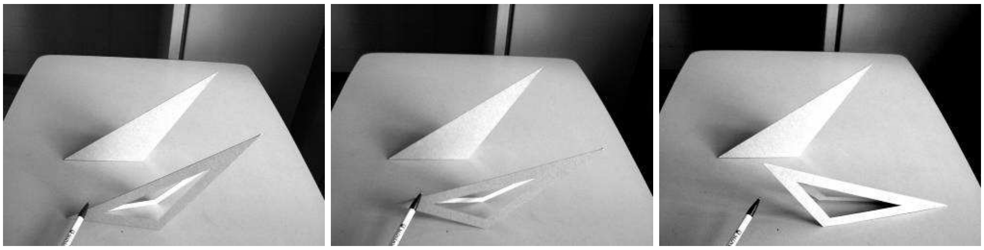


Figure 4: Hands-on experiments with cardboard triangular figures confirm the results of virtual experiments. Solid figure stays balanced while the hollow one tumbles down when released.

Conclusions

The proposed educational research for freshman students combines the creative studies of distinct topics in physics and mathematics as well as in engineering and computer science—all in a single project. After conducting virtual experiments, students pose hypotheses that are later tested by them during experiments with the corresponding real objects that they construct. The mathematical analysis of balance properties of the geometrical bodies investigated in virtual experiments as well as the further construction of various monuments and pedestals provides students with an extensive experience on how useful and efficient mathematics is when applied to various physical and engineering problems [11].

Students discover, in particular, that if an object has a simple geometrical shape (triangular, conic, etc.), the physical definition of the center of mass leads to a simple geometrical description of the center's position. This triggers their curiosity to investigate further the complexity of mathematical formulas that must be embedded in the computer program in order to properly depict on the computer screen the position of the centers of mass. Moreover, by analyzing how the computer program works, students gain better understanding on a variety of ways mathematical formulas are used in computer modeling—to model physical laws, to depict graphics, etc.

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