Teaching Error Theory Using Virtual Data

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Undoubtedly, the best teacher of the error theory is experimental practice. However, practice has three seriously weak points: 1) Practice is expensive and time consuming. It is hard to provide opportunities for all students to design and repeat experiments as much as necessary. 2) Real experiments cannot be controlled completely. Such systems have too many parameters that make all errors to follow Gaussian profile. 3) Parameters of a real system cannot be manipulated easily. Hence, we cannot design "pathological" cases, very important for educational purposes. Relatively simple way to overcome such difficulties is introduction of virtual experiments. Virtual experiment can be defined as computer program that simulate real experiments along with pseudo-random variation of measured values according to postulated statistical distribution. Such experiments appear to be fertile area for investigation of error distributions, mutual dependency of system components and testing of theoretical models. Correct estimation of an error is equally important as correct estimation of mean value. Only "ordered pairs" gives a meaning to a measurement and that is the point where science education usually fails. Formula for an error of complex system based on total differential gives a good prediction of output error only if relative errors become small and if all components happen to be independent. Virtual experiments can help developing of research intuition on such systems. Further, modeling and search for relationships between parameters can achieve higher level of reliability if prove them on virtual variants of the real experiments.

Introduction

Idea presented in this paper emerged from mutual interaction between participants of physics seminars at Petnica Science Center [1,2,3]. Experiments exhibited in this paper are a part of the Learning through Research (LTR) educational model [2,4] realized at Petnica. Target group for this educational approach are extraordinary gifted and motivated high school students already familiar with elements of experimental physics, higher mathematics and programming. Practically, this paper is a concept for a series of lectures whose depth and extent depends only on students' motivation. A lot of calculations presented here were realized through homework proposed to students.

Unlike the traditionally defined curricula, learning through research has a program not specified in terms of what is to be taught, but rather in terms how it is to be taught. LTR is a model that enables two important moments in the process of knowledge acquiring. The first one is an opportunity to "discover". A fact or conclusion emerged as a result of exploration becomes unforgettable as well as a pathway. This route is a major source of research joy and excitement, but a way of much more efficient learning also. The other important moment of LTR is possibility of implementing new science ideas and techniques in order to confirm well-known fact or principle. Results in students' research surely don't have to be "numbers" seen for the first time. Seeking new methods for establishing the same old fact is research as well.

One of the most important problems of physics education is invoking of measurement and error concepts. Intensive laboratory work and analysis of results are equally important parts of the solution. LTR requires a lot of critical thinking, so great attention is paid to the nature of results' analysis itself.

Computers enabled not only new branches of physics, but also new approaches in physics education. Somewhere between experiment and theory emerged numerical physics and that area exhibited enormous potential for extra-curricular learning. Results of measuring in numerical physics have the same attributes as those obtained through classical experiment. However, numeric has one great advantage – computers can perform enormous number of "experiments" and that enables us to see what would happen if we could make a million or a billion measurements. This shift of limits is very important for educational applications.

Virtual measurements

Before we introduce virtual measurements, let's do a simple, classical measuring. Let a simple pendulum swing and measure single periods of its oscillations. Suppose that we have measured ten periods and got following set of values T[s]={1.47, 1.51, 1.44, 1.37, 1.44, 1.47, 1.56, 1.50, 1.58, 1.40}. Ten numbers are more than enough for calculating mean value and standard deviation, so we can present obtained result as T=1.47±0.07 s and that would be the end of an experiment. However, the end of real measurement can be the start of a different one, virtual.

If we choose to use standard deviation as a measure of uncertainty, that way we implicitly assume Gaussian profile of the distribution which is, fortunately, valid for the great majority of different measurements. In other words, we assume that all measured values are coming from a random number generator with normal distribution. It would be nice if we could show students that real measurements of the period really form normal curve, however, that would require hundreds or thousands of measurements. That would be too much for a demonstration. If we make a virtual stopwatch instead, we could make millions of virtual measurements easily. The only difference is that in the latter case we as teachers know exact values of mean value ($\langle T \rangle$) and standard deviation (σ) for our series of virtual measurements doesn't matter how long it is. That could be a nice advantage for a demonstrator.

One of important things about measuring that we would like to show to students is that mean value and standard deviation converge and that our uncertainty about their values is decreasing in a certain way as we increase number of measurements. Naturally, it would take too much time to demonstrate that using real measurements. Look at Figure 1. There is presented how standard deviation depends on number of measurements (for the series of ten measurements given in our example). It is obvious that we cannot be sure that standard deviation converges at all until we make many more measurements.

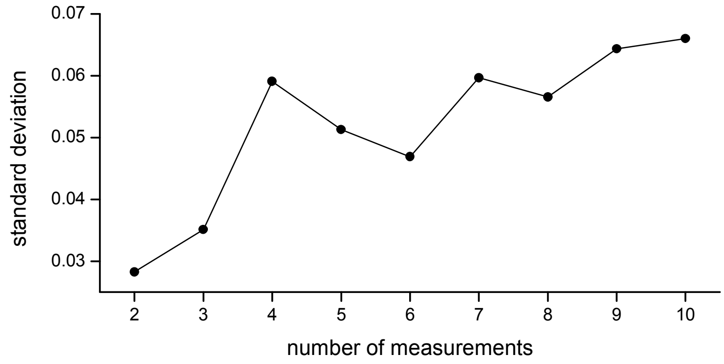

Figure 1: Standard deviation depending on number of measurements

If we want to find out more about the distribution of period values of that exact pendulum, we have to make much more of measuring. However, if we want to investigate certain properties of distributions and errors *per se*, we can use virtual measuring instead. We already assumed that our measurements are samples from normal distribution determined by $<T>$ and σ. We can simulate measuring process by generating pseudo-random numbers from the same distribution. A lot of mathematical software has implemented functions for sampling from normal distribution. In MATLAB, for instance, that function is called randn. At Figure 2 we can see an example how to generate new results obeying the same distribution as for the first ten.

```
T=[1.47, 1.51, 1.44, 1.37, 1.44, 1.47, 1.56, 1.50, 1.58, 1.40];    % ten measured periods
Tm=mean(T);                  % the mean value
Td=std(T);                   % standard deviation
N=10000;
Tv=Tm+Td*randn(1,N);         % array of 10000 values for virtual measurement
```

Figure 2: MATLAB code for generating of virtual results

If we want to see what is alike distribution of values for great *N*, for instance 10000, or to see whether standard deviation really converges, we would have to measure period of a simple pendulum for many hours and that would be hard and boring task. Generated "pseudo-results" simulate real experiment and we can do further investigation. Figure 3 shows a part of histogram for 10000 "pseudo-measurements". Obviously, fluctuations are less exhibited and experimental distribution looks more like expected theoretical curve. Of course, there is no reason why σ should behave in the same manner for yet another virtual experiment. If we want to discover general behavior of σ value, we should find average σ for an ensemble of different experiments.

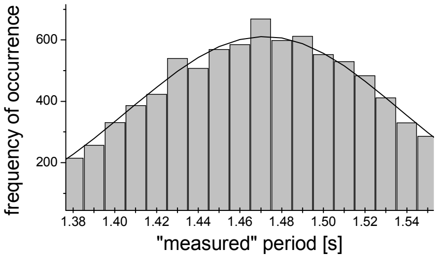

Figure 3: Histogram of virtual measurements and expected theoretical curve

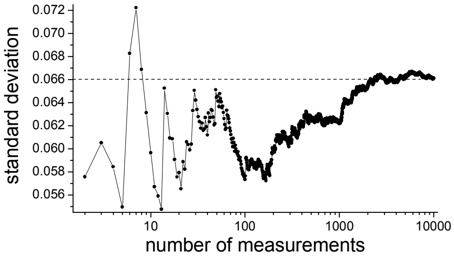

Figure 4: Standard deviation depending on number of measurements

The mean value error

Investigation of general behavior of the mean value and standard deviation demands more than one virtual experiment. The first of all, we would like to show students that uncertainty of the mean value decreases with number of measurements, but such conclusion can not be inferred from a single experiment. MATLAB code given at Figure 5 demonstrates how standard deviation of a set of different virtual experiments mean values depends on number of measurements, i.e. iterations. Further, such "investigations" can lead students to "discover" relations usually presented as a fact that can be inferred from a theory. For instance, let's see what we can get from the example shown at Figure 5. Results are given on the log-log scale at Figure 6. Undoubtedly, we get a power law. The slope of the graph, in this case -0.51±0.02, gives us the power exponent. That means that uncertainty of a mean value decreases as $N^{-0.5}$. When students "discover" that relationship, they are open and ready to learn about the standard error.

```
n=100;              % n represents number of experiments.
Tv=zeros(n,N);      % Instead of one, here we have n different arrays of virtual results.
Tvm=zeros(n,N);             % matrix of the same size for successive mean values
for i=1:n
   Tv(i,:)=Tm+Td*randn(1,N);
   for k=2:N
      Tvm(i,k)=mean(Tv(i,1:k));    % This is the simplest way to compute successive mean
      end                              % values, but very far from optimal.
   end
Tvmd=std(Tvm);      % stand. dev. of mean values for different experiments with the same
                               % number of measurements
loglog(Tvmd,'.-');  % plot on the log-log scale
```

Figure 5: MATLAB code for computing mean values and its standard deviation

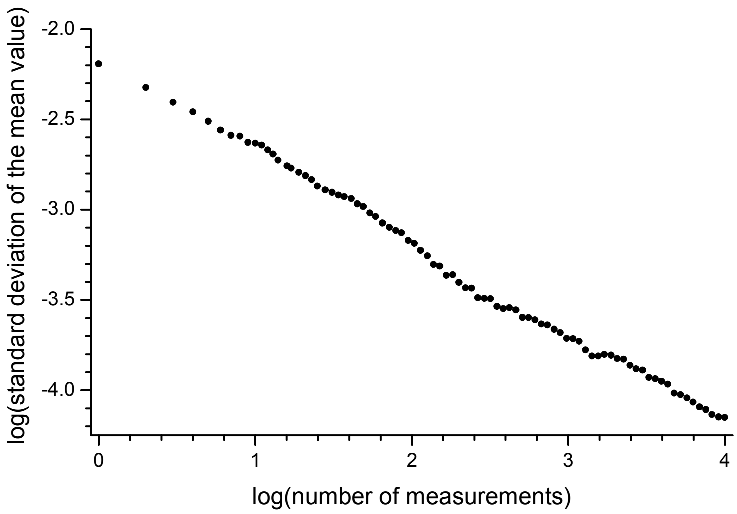

Figure 6: Standard deviation of the mean value vs. number of measurements

Indirect measurements

Virtual experiments are even more useful in indirect measurements. Virtual measurement cannot replace real, just to simulate it using already determined parameters. However, indirect measurements are the result of calculation from one or more direct measurements, so we have the same prerequisites for real and virtual measurement – correctly made direct measurements and relationships between values that enable us to compute the value that cannot be measured otherwise.

Gravitational acceleration cannot be measured directly if we are equipped with a stopwatch and a ruler only, but we can measure periods for different lengths of simple pendulum and hence compute the slope of the graph of the square of the periods against length. It is trivial to apply linear fit on experimental results and obtain the slope. More serious problem is how to compute the slope's error. Unfortunately, majority of available mathematical software do not include information about the error in the least square algorithm and therefore we get unreliable or totally useless estimation for errors of fitting parameters. Problem is even bigger if distribution of measurement values does not obey Gaussian distribution. Using virtual experiments we can determine how individual errors influence final result in all these cases.

l [m]	Δl [m]	T [s]	ΔT [s]	T^2 [s^2]	ΔT^2 [s^2]
0.54	0.01	1.47	0.07	2.15	0.2
0.80	0.01	1.79	0.07	3.20	0.25
1.04	0.01	2.05	0.08	4.20	0.35

Table 1: Lengths, periods and accompanied errors for a simple pendulum

In Table 1 are given values for periods, pendulum lengths and accompanied errors as an experiment example. Our task is to compute *g* and its uncertainty using a virtual experiment.

In this case, virtual experiment samples pseudo-results, for both co-ordinates, from estimated distributions and determines fitting parameters (Figure 7). The mean value and the error of parameters can be obtained statistically. An algorithm for determining gravitational acceleration and its error is given at Figure 9.

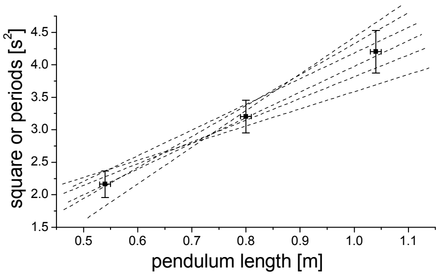

Figure 7: Possibilities of linear fit through points with significant experimental errors

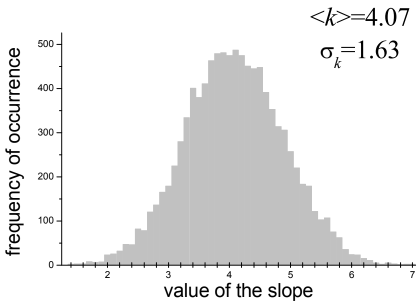

Figure 8: Histogram of slopes from Figure 7

```
x=[0.54, 0.80, 1.04];              % points' x values
y=[2.16, 3.20, 4.20];              % points' y values
ey=[0.2, 0.25, 0.35];              % y-axis errors
ex=[0.01, 0.01, 0.01];             % x-axis errors
for i=1:10000                      % 10000 repeats
        vy=y+randn(1,3).*ey;  % varied y values
        vx=x+randn(1,3).*ex;  % varied x values
        kn=polyfit(vx,vy,1);       % linear fit
        sl=kn(1);                  % a slope
        k(i)=sl;                   % array of slopes
        end
ak=mean(k);                        % average value of slopes
dk=std(k);                         % standard deviation of slopes
hist(k,20);                        % histogram of slopes
g=(2*pi)^2/ak;                     % value for g
dg=dk/ak*g;                        % value for Δg
sprintf('g=%1.1f±%1.1f',g,dg) % display final result as g±Δg
```

Figure 9: MATLAB code for determining distribution of virtual values of *g*

Histogram of virtual slopes clearly visualizes distribution of that parameter. The same routine we can use to calculate the intercept or coefficient of correlation.

Conclusions

The error theory is a tale about small students' discoveries and better understanding of natural phenomena. There are, obviously, many things concerning correct use and interpretation of results that cannot be demonstrated in the classroom because of the lack of the time. Using of virtual measurements can speed up the learning process significantly.

It is also important to notice that there exist a lot of experiments that we cannot easily perform in schools for reasons other than lack of time, like radioactive decay. Simple virtual experiment could compensate such a shortage. Ultimately simple mechanism of radioactive decay, for instance, allows us to simulate the process by typing a few lines of code only in front of our students. That is real advantage of computers in a classroom.

References

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