Friction Holding the Climber: An Experimental Example from Physics in Sports

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Climbers use a so called belay devices to support each other and to help slowing down a descending companion. Sliding friction plays an essential role in such devices. A model of the belay device is that of a rope wounded around a cylindrical rod. It turns out that due to friction between the rope and the rod the ratio between the tensions of the rope at two opposite sides of the winding increases exponentially by the angle of winding. An experimental apparatus is presented that allows for dynamical computer-based force measurement and permits to explore the exponential relation between the two forces. Furthermore, an interesting interplay between static and dynamic friction can be observed, which is typical for example in violin playing. The exercise proved to be a very fruitful open-ended laboratory exercise. Students are challenged to construct an appropriate experimental setup. The computer-based measurements are suitable for students training in data acquisition and analysis. Furthermore, it allows interesting insights into the physical phenomenon.

Introduction

To motivate students, bridges are needed between formal knowledge and everyday experience. The application of achieved formal knowledge to explain everyday phenomena is crucial for gaining motivation and interest in physics. Examples of connection between physics and other natural sciences, such as biology or chemistry are seldom used in teaching physics. We are presenting an example from physics in sports that can be useful in stirring students' interests and motivate them for either qualitative reasoning or to use a more formal approach to understand the physics behind the phenomenon.

Climbers use the so called belay devices (Fig. 1), which help them to support each other and to slow down a descending companion. One end of the rope which passes through a belay device is attached to the climber while the other end is held by belayer. The sliding friction supplied by a belay device helps the belayer to hold the descending climber without huge effort. A rope is attached to the device in a particular way; however the basic principle is that the rope is wound around the cylindrical rod of the device. Another example where friction is made useful by a similar principle is found in rope windings on sailing boats and ships. In contrast the friction may be very annoying like for example when cables are used on a bicycle to pull on a derailleur or a brake. The cable is lead through a cable housing which is often Teflon coated to minimize friction. If the cables are bent the friction would increase drastically thus impairing on bicycle's functionality.

We have made a simple model of a belay device equipped with computer-based force measurement. The theoretical predictions of the belay device behaviour can be tested by means of two force sensors connected to an interface. The forces can be measured in dynamic regime while the rope is sliding. Interesting insights into the interplay between static and dynamic friction occurring during the sliding process can be revealed. The same stick-slip phenomenon is responsible for example in producing sounds by a violin.

At Faculty for Mathematics and Physics in Ljubljana we employed the experimental problem of analysing the behaviour of a belay device model as an exercise for first year physics students attending a course called "Project laboratory" [1]. The course takes place three times per three hours and there is additional time reserved for students to check out the literature, confront ideas etc.
The problem was to construct the apparatus that would allow verifying the exponential relation between the rope tensions at the two opposite ends. It turned out that the exercise was very fruitful in stimulating students' ideas and creativity in testing the experiment and analysing the results. Different designs where considered, tested and compared by students [1].

Figure 1: Different kinds of belay devices used in climbing.

Theory

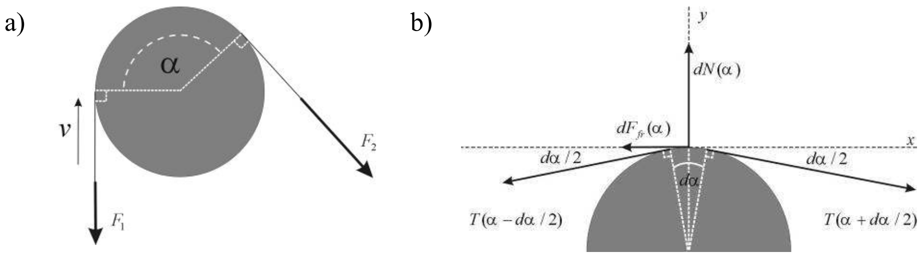

Figure 2: Scheme of the experiment: a) With F_1 and F_2 we denote the forces at two opposite ands of the rope, which is wound around a cylinder through an angle α. The rope slides with constant speed *v*. b) Small section of circumference where the tension forces $T(\alpha)$, friction force $dF_{fr}(\alpha)$ and normal component $dN(\alpha)$ are in dynamic equilibrium.

The rope is wound around a cylinder through angle α. Force F_2 is applied at one end of the rope while force F_1 is applied on the opposite end to let the rope slide at a constant speed (Fig. 2.a). We consider a dynamic equilibrium of forces. We take F_2 to be larger than F_1 in order to compensate for sliding friction between the rope and the cylinder.
In order to see how the ratio of forces at the two opposite ends of the rope varies with angle α we must analyse how the tension of the rope $T(\alpha)$ varies along the rope winding. The tension and friction forces change their direction along the cylinder circumference. We will focus on a very small section of the circumference spanned by angle $d\alpha$ where the tension force variation and the friction force contribution are very small (Fig. 2.b). In order to write down the Newton's first law of motion we account for the equilibrium of forces separately in tangent direction denoted by *x* and in radial direction denoted by *y*. The equilibrium in *x* direction is obtained when

$$dF_{fr}(\alpha) = T(\alpha + d\alpha / 2) - T(\alpha - d\alpha / 2) \tag{1}$$

and in y direction when

$$dN(\alpha) = \left[T(\alpha + d\alpha/2) + T(\alpha - d\alpha/2)\right] d\alpha/2\,. \tag{2}$$

Here we have approximated $\sin(d\alpha/2) \cong d\alpha/2$ and $\cos(d\alpha/2) \cong 1$. We can see from (Eq. 1) that the rate of tension change $dT(\alpha)=T(\alpha+d\alpha/2)-T(\alpha-d\alpha/2)$ varies according to $dF_{fr}(\alpha)$. But the friction force can be expressed by the normal component $dN(\alpha)$ as

$$dF_{fr}(\alpha) = k \cdot dN(\alpha)\,. \tag{3}$$

If we consider that the rate of tension change is very small the sum in (Eq. 2) can be approximated by considering that $T(\alpha+d\alpha/2) \cong T(\alpha-d\alpha/2) \cong T(\alpha)$. The (Eq. 2) thus tells us that the normal component $dN(\alpha)$ is proportional to the tension $T(\alpha)$ and (Eq. 3) can be rewritten as

$$dF_{fr}(\alpha) = k \cdot T(\alpha) \cdot d\alpha\,. \tag{4}$$

Equation (Eq. 1) can now be written in the form of differential equation for the tension force

$$dT(\alpha) = k \cdot T(\alpha) \cdot d\alpha \tag{5}$$

and integrated from $T(0)=F_1$ to $T(\alpha)=F_2$

$$\int_{F_1}^{F_2} \frac{dT}{T} = \int_0^{\alpha} k \cdot d\alpha \tag{6}$$

to give the exponential angle dependence of the ratio between the forces applied at the opposite sides of the rope

$$\frac{F_2}{F_1} = e^{k\alpha}\,. \tag{7}$$

We may explore the exponential relation (Eq. 7) by setting any of the three variables constant. If either F_1 or F_2 is constant we have an exponential relation between force and angle, but if α is constant there is a linear relation between the two forces. In the first case when F_2 is constant, taking for example the weight of a climber, we see that F_1 decreases exponentially with angle of winding α meaning that a person can hold a very heavy weight just by using a sufficiently large angle of winding. If the angle of winding is constant the ratio between F_1 and F_2 is fixed and depends on the coefficient of friction. The rougher is the rod the better for holding a heavy weight.

Experimental setup

The experimental exercise consists in exploring the exponential relation of forces F_1 and F_2 at the opposite ends of the rope in dependence of the angle of winding α. The relation can be tested if the force ratio is measured at different angles. But to test a linear relation of forces in case of fixed angle we have to measure one of the forces while the other is changing. We do this by continuously varying the tension of the rope while both of the forces are being measured. This way the linear relation is tested by a single measurement. By repeating the measurement at different angles of winding the exponential angle dependence of the force ratio is tested. Coefficient of friction can be measured from the slope of F_2 vs. F_1 which equals $\exp(k\cdot\alpha)$ at a fixed angle or from the slope of $\ln(F_2/F_1)$ vs. α when the experiment is repeated at different angles.

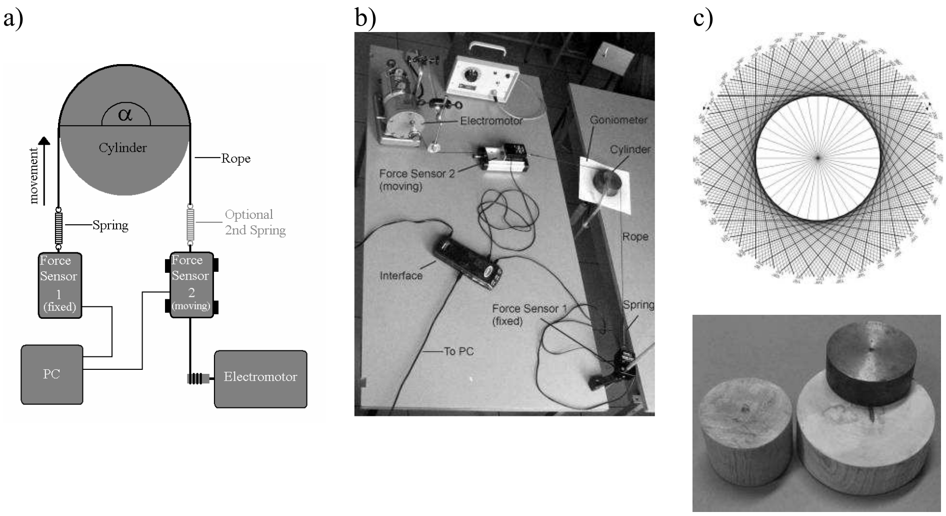

Figure 3: Experimental setup and accessories: a) Scheme of the setup, b) Photo of the setup, c) Goniometer and different cylinders.

Our experimental setup allows for dynamic measurement of forces F_1 and F_2 by data logging (Fig. 3.a, b). One end of the rope is attached to a linear spring which is then attached to a fixed force sensor. The other end of the rope is pulled with uniform speed by a stepping electric motor. The motor maintains a constant speed by increasing its torque while the spring is being stretched. To measure the force supplied by the motor a movable force sensor is placed on a frictionless cart between the rope and the motor. This way the measurement is made dynamic as the rope is allowed to slide. The time course of the two forces is measured during an experiment. Many corresponding pairs of F_1 and F_2 are collected in a single experiment which would not be possible for instance by using a hanging body with a fixed mass.

To measure the angle easily, we designed a goniometer that is simply printed on a paper and fixed on a table together with a cylinder (Fig. 3.b, 3.c). We used Lab Pro interface, Vernier dual range force sensors and Logger Pro software for data processing.

If an additional spring is attached to the movable sensor a stick-slip behaviour can be observed (Fig. 6). The stick-slip behaviour presents an intriguing surprise that is interesting to discus qualitatively and analyse quantitatively by a mathematical model.

The stick-slip behaviour analysis

The stick-slip behaviour can be observed if an additional spring is used (Fig. 3.a). In order to analyse the behaviour (Fig. 6) we denote the coefficient of dynamic friction by k_d and the coefficient of static friction by k_s. While the tension in the rope continuously increases we observe that the rope keeps alternately sticking and slipping upon the cylinder. The time course of F_1 and F_2 reveals that F_1 changes in discrete jumps (Fig. 6.a). While F_1 is constant the force F_2 keeps increasing continuously until F_1 jumps to a higher value. At that moment

F_2 abruptly decrease and restart increasing again from a lower value. We can observe this more explicitly by looking at F_2 vs. F_1 graph (Fig. 6.b). We can see that F_2 increases from its minimum value $F_{2\min}$ to its maximum value $F_{2\max}$ only at discrete values of F_1. From the following analysis (Eq. 10) we see that the interval $F_{2\max}$ -$F_{2\min}$ increases linearly with F_1.
Jumps occur when the rope slips over the cylinder. So the minimum of F_2 at a given F_1 can be expressed by the coefficient of dynamic friction

$$F_{2\min} = e^{k_d \alpha} F_1 . \quad (8)$$

But when the rope sticks on the cylinder the static friction increases to balance the increasing force F_2. While the force F_1 is constant the force F_2 increases till the coefficient of friction reaches its maximum value k_s. Thus at given F_1 the maximum force F_2 would be

$$F_{2\max} = e^{k_s \alpha} F_1 . \quad (9)$$

The static and dynamic coefficients of friction give the span of the force F_2 at a given value of the force F_1 as

$$\Delta F_2 = F_{2\max} - F_{2\min} = (e^{k_s \alpha} - e^{k_d \alpha}) F_1 . \quad (10)$$

Results

We measured the time course of the forces F_1 and F_2 during pulling of a dry and a wet rope. When the forces are measured at a fixed angle of winding we can observe that the puling force F_2 increases together with the opposing force F_1 (Fig. 4.a), however the ratio between the two forces is constant and exponentially depends on the coefficient of friction. By measuring the angle of winding and the ratio of the two forces or the slope of F_2 vs. F_1 (Fig. 4.b) we get the coefficient of friction as $k=\ln(F_2/F_1)/\alpha$. In case of a dry rope wound through 80 deg the measured coefficient of friction was 0.26.
By repeating the experiment at different angles we can examine the forces ratio as a function of angle (Fig. 5). The exponential relation is tested and from an exponential fit or from a linear fit of $\ln(F_2/F_1)$ versus the angle of winding we get again the coefficient of friction. For a dry rope the value was again 0.26 while for a wet rope the coefficient of friction was 0.30 which is for about 15% larger than in case of a dry rope.

a) b)

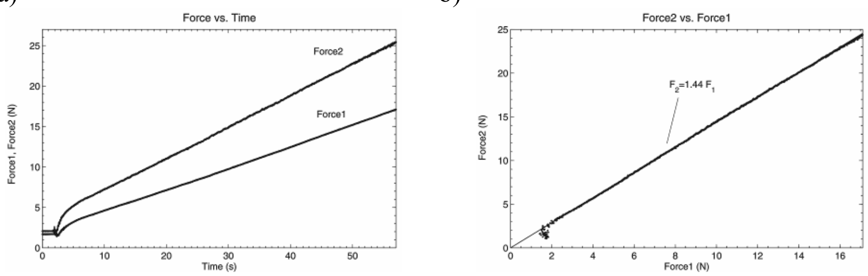

Figure 4: Experimental data for dry rope at α=80 deg: a) Time course of the two forces and b) F_2 vs. F_1 plot.

By using an additional spring the stick-slip behaviour was observed. Forces increase in a stepwise fashion (Fig. 6.a). The force F_2 increases from a minimum to a maximum value at a definite value of F_1 (Fig. 6.b). These minimum and maximum values increase linearly with F_1 and consequently F_2 range increases linearly with F_1 as well (Eq. 10). From F_2 vs. F_1 plot one can calculate both the coefficient of dynamic and static friction. The estimated values for a dry rope were k_s=0.28 and k_d=0.21 (Fig. 6). We can infer that in the absence of the additional spring the stick-slip behaviour occurs in very small steps due to small elasticity of the rope and thus it is very difficult to be observed.

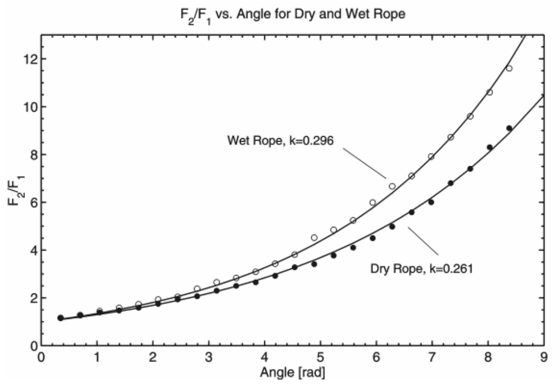

Figure 5: Ratio of the two forces vs. the angle of winding for a dry and a wet rope.

a) b)

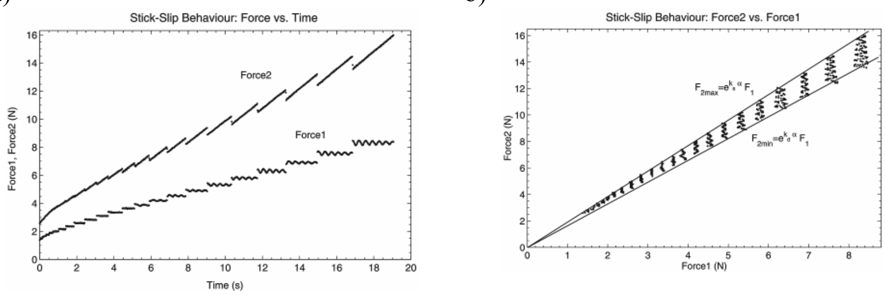

Figure 6: Stick-slip behaviour for a dry rope at α=130 deg: a) Time course of the two forces and b) F_2 vs. F_1 plot.

Discussion

We propose some guidelines to lead students through analysis and to outline a qualitative discussion. Students should consider the forces that act on the rope at any separate fraction of the rope along the winding and realize that the tension in the rope varies from the value of F_1 to the value of F_2. They should be encouraged to explain the origin of variations in rope tension. The equilibrium condition and the Newton's first law of motion should be considered in order to realize that tension variations have to be compensated by friction.

The friction force acts tangentially and depends on the normal component at any point along the winding. Students can infer about the course of normal component along the winding. It can be seen that the normal component per unit length depends on the tension of the rope and on its curvature. Thus, the students are lead to realize that the friction is proportional to the

tension at a particular point along the rope winding. So the increase of the tension along the winding is proportional to the tension itself.
This observation could be illustrated by some analogies like it is for example the course of capacitor discharge or the temperature increase of a thermometer. Whenever the rate of change of a variable is proportional to the value itself the course is exponential. If the temperature difference for example decreases in proportion to the temperature difference itself and to a fraction of time the temperature will decrease exponentially with time. Similarly does the tension increase/decrease exponentially with angle as the tension increases/decreases in proportion to the tension itself and to a fraction of angle.

Conclusion

Although the presented example may seem like another dry exercise from physics textbooks it can be illustrated with many examples from everyday life. The examples of exponentially increasing friction force are intriguing and stimulate analysis aimed to understand the phenomenon. The behaviour of a sliding rope can be analysed quantitatively by computer-based force measurements, while graphical representation of the results could considerably contribute to fruitful qualitative discussions.
The example can be presented in qualitative demonstrations or studied in students' project laboratory exercises. The data logging gives a quick and detailed insight which can stimulate students' analysis and enable the teacher to demonstrate the phenomenon efficiently.

Acknowledgment
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References

[1] http://student.fmf.uni-lj.si/fiz/projlab/