The Force between Electric Charges and the Origin of Magnetism

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We investigate the force between two electric charges and explore the origin of the magnetic force. We show that magnetism is a pure motional consequence of the electric force. From the consideration, special relativity arises naturally.

Introduction

From a pedagogical point of view the traditional way of teaching special relativity (SR) contradicts several basic teaching strategies. Normally, we would base teaching on accessible observations and conceptual understanding related to daily experience using concrete concepts only. As a second step, one would perform measurements and quantify mathematically. Finally the search for connections with other phenomena, interpretations and model makings take place where the introduction of abstract concepts might be necessary.

In the standard teaching of SR the above pedagogical strategy is more or less abandoned. At the starting point, we make extensive use of abstract concepts such as fields, light, time and space as if these were known a priori. The time concept is based on light pulses sent back and forth. The propagation time of these is taken as a time measure, a time definition which is spurious. In fact, a natural and conceptual time definition is lacking in SR which is a key reason for the general confusion of the real meaning of SR.

The Michelson-Morley experiment is the only experimental basis, used for showing light speed invariance. The experiment is hard to perform and difficult to interpret since it involves the abstract concept of light. In fact, the experiment is hardly ever done by the students themselves. It also gives the wrong impression that light speed invariance is the origin of SR, since it is really the fact that interactions take time.

As a consequence SR is filled with several thought experiment, such as space ship travels and other phenomena no one has ever experienced. In the end, teachers therefore very often emphasize that SR has no influence on real life. However, nothing could be more wrong since it actually is the condition for life.

We therefore propose a method of introducing SR based on what is actually observed, emphasizing the use of concrete concepts. By making clear that the sole observables in nature are objects, their relative distance and their relative motion we obtain natural definitions on space and time. Our recipe for SR follows the agenda:

1. Introduce electric and magnetic force
2. Introduce model for electric current
3. Identify the pairwise interaction between identical charges
4. Introduce the principle of relative motion
5. Identify motional effects (magnetism, time dilation)
6. Interpretations

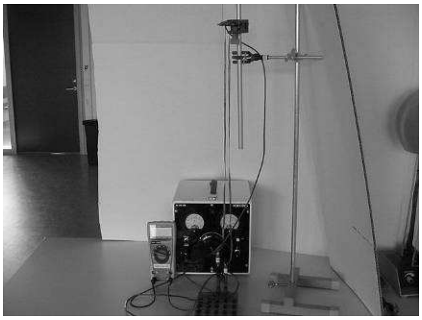
 

Fig. 1: Charged balls on an air flow table and parallel conductors for studying electric and magnetic forces.

1. Forces

The observation of electric and magnetic forces constitute the phenomenological basis for our approach to SR. The measurements of these are tedious but straightforward, easily accessible to students at any level, fig. 1.
In this way the Coulomb electric force

$$\overline{F}_e = \frac{qq'\hat{R}}{4\pi\varepsilon_0 R^2}$$

and the magnetic force between parallel current-carrying conductors

$$\overline{F}_m = \frac{\mu_0 II'}{2\pi R}\hat{R}$$

are measured and verified. The magnetic force law can be written as

$$\overline{F}_m = \frac{II'}{\varepsilon_0 c^2 2\pi R}\hat{R}$$

so that the measurement will determine the quantity c which we later recognize as speed of light.
With the help of Biot-Savart law and the Lorentz force the magnetic force between current elements Jdτ of the conductor is

$$d\overline{F}_m = \frac{1}{4\pi\varepsilon_0 c^2}\frac{\overline{J}d\tau \times \left(\overline{J}'d\tau' \times \hat{R}\right)}{R^2}$$

where J is current density, i.e. current per area, and dτ is a volume element.

2. Model for electric current

Traditionally, our primary model for electric current is the flow of charged particles, introduced very early in school. Fig. 2 shows our model corresponding to the flow of electrons in the current-carrying conductor. Of course, this is a highly idealized description, taking into account only the drift motion of the electron and neglecting any quantum mechanical effects. Still, relativity is based on this simple picture of nature as will be shown.
In figure 2 we note the electric force, not appearing in case of electrically neutral conductors.

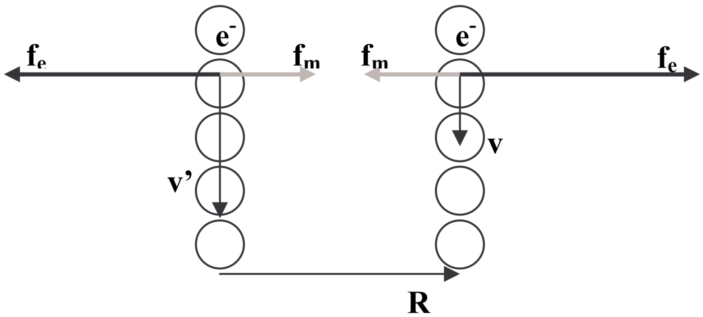


Figure 2: Two parallel electron beams with speeds v' and v.

3. The pairwise interaction

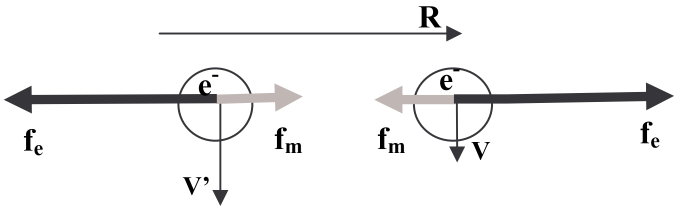


Figure 3: Electric and magnetic forces between charges in motion.

The basis of physics is the pairwise interaction. In any area of physics, the system is considered as built up from constituents. The total force on one of the objects is the sum of forces from all the other. This holds true also in non-linear theories such as gravity and strong interactions, taking into account so called feedback effects of the pairwise interaction. In case of electromagnetism, we can easily identify the pairwise interaction as shown in fig. 3. We maximize the simplification by choosing the same speed v for the two objects.

Considering the current as a flow of particles, the current density $\bar{J} = \rho\bar{v}$, i.e. charge density times drift velocity of the electrons. Identifying ρdτ as the charge of one electron q, the magnetic force on the primed (right-hand) charge of fig. 3 is

$$\bar{F}_m = -\frac{1}{4\pi\varepsilon_0 c^2}\frac{v^2 qq'}{R^2}\hat{R}$$

4. Principle of relative motion

As one of the two postulates for SR Einstein chose the principle of relative motion. Einstein's main argument for this principle was based on the induction phenomena. When a magnet approaches a closed conductor, the same current is generated as when the conductor approaches the magnet. This insight might seem trivial but it gives some rather striking consequences.

Fig. 4 illustrates the principle w.r.t. the pairwise interaction.

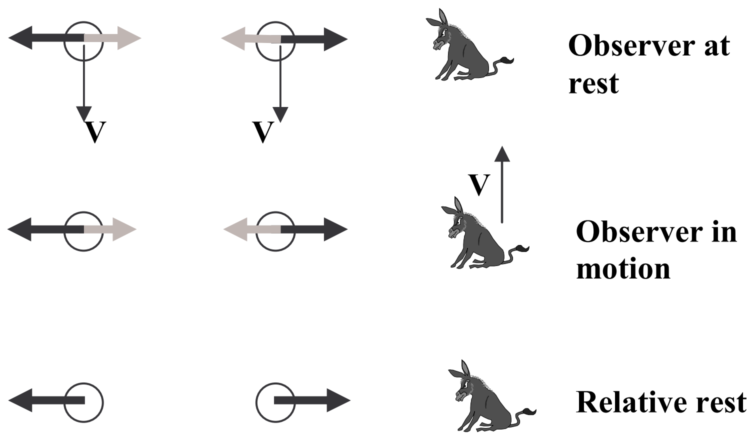


Figure 4: Principle of relative motion. The grey arrows are magnetic force, the black electric force.

Because of the principle of relative motion the force situation must be identical in the upper and the middle figure. Thus, the observer generates the magnetic force purely by putting herself in motion.

Therefore, magnetism is a pure motional effect and shouldn't be called a force. Compare our treatment of centrifugal and coriolis effects: being motional effects, i.e. they are generated due to the motion of the observer, we don't call them forces but rather 'effects'. Accordingly, the magnetic force doesn't exist. There is only one force, the electric one, and its magnitude varies depending on the motion of the charges. As a consequence, it should not be possible to introduce a magnetic field, forcing us to reconsider the traditional Maxwell model of the electromagnetic wave.

Furthermore, it is interesting to note that the magnetic effect between moving charges is a well known effect in particle accelerators. Close to the speed of light the magnetic effect balances the electric force which facilitates the stabilization of the beam. This experimental fact is important since it verifies that it is not the relative motion between the charges that causes magnetism but rather their motion relative an observer. This remarkable fact was not realised before Einstein.

5. Motional effects

Our concept of time and space are heavily inherited from authorities, efficiently introduced in our language. However, people working with human sciences such as art and literature often discuss and question the concept of time and space. We physicists ought to treat these concepts more carefully.

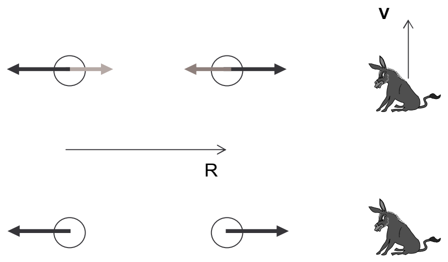
Since what are basically observed in science are objects, their relative distance and relative motion, it is natural to interpret space as distance between objects and time as motion. Both distance and time needs some reference and are therefore relative and also connected.

Noether's symmetry theorem verifies this notion of time and space, stating that energy and momentum conservation arise due to symmetry in time and space respectively.

For each object, energy and momentum is conserved only if there is no active force, equivalent to unchanged motion and distance. On the other hand, an active force changes energy-momentum of the object and accordingly space-time leading to the basic idea of general relativity, i.e. the curved space-time.

Noether's theorem is therefore just a reflection of the above definition of time and space.
Now, coming back to the pairwise electromagnetic interaction, fig. 4, we note that due to the magnetic effect the transverse motion slows down when observed in motion compared to relative rest. Using our concept of time, a slow down in motion means a slow down of time flow, called time dilation in SR. In a teaching situation one might want to make use of the film example here. A recorded film can be played at a slower speed giving a slow motion effect interpreted generally as a slow down of time, convincing that our concept of time is naturally connected to motion.
It is now straight-forward to derive time dilation, the essence of relativity, using Newton's force law, modified for the relativistic notion of time, fig. 5.

Time is motion in space. Observer in motion experience time T, observer at rest T_0



$$m\frac{\Delta\left(\frac{\Delta\bar{R}}{\Delta T}\right)}{\Delta T}=\frac{1}{4\pi\varepsilon_0}\frac{qq'}{R^2}\left(1-\frac{v^2}{c^2}\right)\hat{R}$$

$$m\frac{\Delta\left(\frac{\Delta\bar{R}}{\Delta T_0}\right)}{\Delta T_0}=\frac{1}{4\pi\varepsilon_0}\frac{qq'}{R^2}\hat{R}$$

m is mass

Assume $\Delta T = \gamma\Delta T_0$ Divide equations =>

$$\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\quad \text{i.e.}\quad \Delta T=\frac{\Delta T_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Time dilation is the essence of special relativity

Fig. 5: Derivation of time dilation

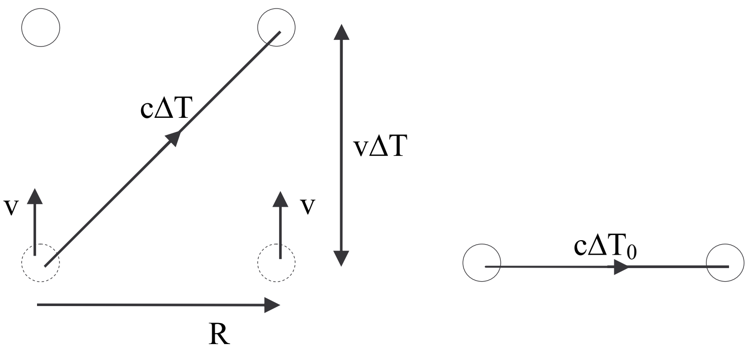
Another remarkable consequence of SR is the fact that nothing travels faster than speed of light c. (In our approach c is just an experimental constant later to be identified as the light speed, see next section.) Although the magnetic force law was measured for low speeds it turns out that it is valid to extrapolate to high speed, verified for example by the accelerator example above. At speed of light then (v=c), the magnetic force equals the electric force and at higher speeds the observer would see an attraction instead of a repulsion. In the film example, this is equivalent of playing the movie backwards giving the impression of reversed time direction and violation of causality. Due to this, we have to postulate that light speed (in vacuum) is the upper limit to possible speeds. A more basic understanding for this astonishing result is not known at the moment.

6. What does all of this mean – an interpretation

Finally, we make an interpretation of our result. We ask ourselves: why does the magnetic effect occur, i.e. what is the origin of the 'magnetic force'? Pedagogically it would be advantageous to let the students come up with ideas at this stage. Having presented the phenomena in the above logical order one would expect that some students really find themselves one of modern science's most important discoveries, i.e. that interactions take time. We even obtain the speed of the interactions in this way.

Fig. 6 illustrates this idea. When charges are moving the effective interaction distance increases, causing the coulomb force to decrease. Time dilation can be obtained by assuming that the motional change of one of the charges is felt by the other charge by receiving pulses of information. A longer propagation time occurs when the propagation distance increases which we experience as a slow down of time. This time concept is of course the conventional one mentioned above and certainly needs careful consideration. In order to achieve the already derived time dilation we have to put the propagation velocity to c, see fig. 6. Furthermore, the propagation velocity cannot depend on the motion of the source since we are forced to use the same propagation speed for both cases. We draw the conclusion that the mediator travels at speed of light independent of the motion of the source. The latter property is characteristic of waves (compare with sound and water waves here) indicating that we deal with a wave motion.

Due to the principle of relative motion it must also hold true that the speed of the interaction does not depend on the motion of the observer. Thus, we arrive at the Einstein principle of light speed invariance, without actually using the concept of light.



$$(c\Delta T)^2 + (v\Delta T)^2 = (c\Delta T_0)^2$$

$$\Delta T = \frac{c\Delta T_0}{\sqrt{c^2 + v^2}} = \frac{\Delta T_0}{\sqrt{1 + \frac{v^2}{c^2}}}$$

Fig. 6: Derivation of time dilation from propagation speed.

7. Summary

Using the electric and magnetic forces as experimental inputs, the naïve model for electric current and the principle of relative motion we may identify the magnetic force as a motional consequence of the electric force. Therefore, magnetism is not a force but rather an effect.

Relativistic effects are the same thing as motional effects which in turn arise due to magnetism. Time dilation results directly from a rather simple analysis of the force between electric charges, using motion as the proper time definition.
The interpretation of magnetism gives at hand that interactions take time and that the mediator travels at the speed of light independent of the motion of its source, a typical property of waves. This is equivalent to Einstein's postulate of light speed invariance which accordingly is a consequence of this analysis. Therefore, the rather intricate experiment of Michelson-Morley is redundant, which was also noted by Einstein.
Finally, we note that this analysis, being free from the concepts of fields and light, is completely independent of Maxwell's field equations.

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