

## WORKSHEETS FOR PUPILS INVOLVEMENT IN LEARNING QUANTUM MECHANICS

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### 1 INTRODUCTION

The paradigmatic role of quantum mechanics (QM hereafter) in the nowadays physical description of the microscopic world and its consequent cultural value are the main motivations for its inclusion in almost all high school curricula [1]. The educational research pointed out the limitation of approaches reconstructing the historical path which led to the recognition of quantization, that is the discreteness of values, observed in many physical quantities of microscopic systems, like energy and angular momentum [2]. Therefore research called attention to teaching-learning proposals directly facing the conceptual elements of QM [2-4]. Furthermore, it also stressed that in QM teaching-learning a significant role is played by the instructional tools (much more than for other curricula innovations) which make explicit and practicable in class activities the conceptual path, warranting some coherence criteria, providing guidelines to teachers in order to build their own personal educational paths [5-9].

The teaching/learning proposal presented during the workshop [10-13], is constituted by a didactic path and several tools for teachers training and in-class experimenting activities, such as: its presentation on the web and on paper, the discussion of conceptual knots which characterise this proposal according to the Dirac disciplinary layout adopted here; the documentation of experiments that constitutes the cognitive anchors for pupils understanding, computer tools in order to realize a palette of ideal experiments where exploring and constructing concepts and ideas, the work-sheets for students and the documentation of results in class activities. [14,15].

We present the work-sheets which are an integral part of the didactic proposal. The work-sheet version presented here emerges as elaboration and re-organization of the work-sheets used and validated during in-class activities. First we present the general worksheet features, then we briefly summarize them in modules. We finally discuss the last worksheet dedicated to build the vectorial formalism of a QM state in detail.

### 2 GENERAL FEATURES OF WORKSHEETS

Worksheets elaborated for pupils activities have been developed according to the teaching physics by inquiry [16] and to the real time laboratory [17]. Moreover these worksheets refer to the results emerged from research conducted the last 10 years [18-21]. The worksheets have been designed on the bases of research

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integrating the didactic analysis, the comparison of different approaches already existing in literature, the cognitive paths of students, thus following a schema quite similar to the Model of Educational Reconstruction [22].

They can be organized in three groups: phenomenological worksheets, thought for the exploration of phenomena about light polarization and their characterization with empiric laws; conceptual worksheets, in which the concept of quantum-mechanic state is built by exploring different interpretative hypotheses and the consequences brought by these hypotheses; worksheets about the formalism, in which students are driven to the mathematical representation of states and observables. Although with important differences, even in structure, and focused on different goals, all worksheets refer to the following general model:

- the issue – the issue is briefly outlined; it constitutes the starting point to work on the conceptual knot which is the worksheet's goal;
- the situation – the problem is contextualized in one or two experimental situations, each of them allowing to overcome a specific conceptual microstep;
- the prediction – for each situation students predict the outcome of the experiment, both in real or ideal situations, about a specific qualitative or quantitative aspect;
- the experiment – students carry out the experiment, or the simple experimental observation, focusing on the specific aspect under investigation, they describe the main features of it, for example reading the real time graph, dividing it in phases, linking phases and processes;
- the comparison – predictions and experimental results are compared in order to recognize differences and analogies;
- conclusions – the elaboration of recorded data and the comparison with the formulated predictions lead to a big picture that allows to generalize the results obtained, and to address new issues.

Most of worksheets are aimed at a single conceptual microstep therefore each of them is composed by 1-2 pages. There are only a couple of them 4-page long and just one is 6-page long. Those worksheets have a complex goal, which is splitted into several microsteps, each of them analyzed in different sections, summarized in a synthesis conclusion of the whole worksheet.

The worksheets are linked together in a logically structured path, which allows to face in an organic and complete way, the main knots of the developed proposal [14,15]. The modular structure of the worksheets also allows a non-rigid use of them, in order to follow different didactic paths, for example paths more focused on conceptual aspects rather than aspects linked to the formalism. (Fig. 1). They are developed keeping in consideration the results of researches about students learning processes and therefore allow to find answers to their typical learning problems, to follow their typical reasoning sequences, to propose main QM knots in a motivating and involving context. They allow, in particular, to face QM in a theoretical orthodox/standard way, but also to explore some



### 3 SYNTHESIS OF THE STUDENTS WORKSHEETS ORGANIZED BY MODULES

#### 3.1 Constructing the phenomenological picture

To operatively construct the concept of polarization as a property of light, we design the following basic worksheets: FenA, FenB, FenC, FenMalus. These worksheets are a selection and a re-elaboration of a set composed by 18 worksheets aimed at exploring light linear polarization in the context of a didactic proposal in physical optic [24].

*FenA* - This worksheet proposes to explore the polarization observing the light transmitted by a polaroid on a OHP. We suggest simple actions (on the OHP: put a polaroid, move it, rotate it, superpose two polaroids, rotate one with respect to the other). For each action students are requested to recognize and record changes in the light intensity transmitted by the polaroids, and to comment these changes. The worksheet is composed by six steps respectively aimed at operatively recognizing: the polarization from the angular variation of the light intensity transmitted by an analyzer (a polaroid rotated around the direction of the incident light); that the light emitted by OHP is not polarized, while the one transmitted by a polaroid acquires a property (is polarized - is prepared); the orientation in space of a polaroid and the polarization of the transmitted light; that the type of light polarization depends only on the orientation of the last polaroid that is crossed (not on the polaroids crossed beforehand); a first formalized representation of polarization with an arrow; the active role of a polaroid in the interaction with light.

*FenB and FenC* - In these two worksheets the phenomenological context of polarization by birefringence is proposed. Using open and closed questions, these worksheets aim at recognizing the main characteristic features of birefringence and to build competence on situations, that we will propose to explore in an ideal context in order to recognize the impossibility to assign a trajectory to a QM system, in which a polarized beam strikes on two aligned birefringent crystals.

*FenMalus* - In this worksheet data are collected. It helps students to carry out measurements in order to build the phenomenological Malus law starting from real data.

#### 3.2 From phenomenology to probabilistic interpretation

A worksheet (ConA), mostly structured with open questions, proposes to analyze ideal situations and re-analyze real situations in terms of interaction of photon beams with polaroids and to give a probabilistic re-interpretation of the experimental results. With a spiral sequence, it allows to make explicit the relation between: the light intensity  $I_t$  transmitted by polaroids, the photon number  $N_t \propto I_t$ , the probability of transmission  $P_t$  of each photon, according to the Malus law.

*Properties of a photon state: mutually exclusive properties and interpretative hypotheses* A set of three worksheets (ConB-C-D) favours the connection between the probabilistic description of phenomena and the QM state construction. It faces

the following conceptual knots: preparation of a QM state, or the measurement as production process of QM systems with well defined dynamical properties; iconographic representation of these properties; recognition of mutually exclusive properties, characterizing a system in orthogonal polarization states (a base of state space); interpretative hypotheses formulation about the nature of a superposition state.

The student chooses between three logically distinct hypotheses proposed as representation of a  $45^\circ$  polarized photon beam:

- A) A beam of photons with the property  $\diamond$  is equivalent to a beam composed on average: half by photons with the property  $\Delta$ ; half by photons with the property \*
- B) A beam of photons with the property  $\diamond$  is equivalent to a beam composed on average: half by photons which simultaneously own the properties  $\diamond$  and  $\Delta$ ; half by photons which simultaneously own the properties  $\diamond$  and \*
- C) A beam of photons with the property  $\diamond$  is composed by photons with the sole property  $\diamond$  which is completely different both from the property  $\Delta$  and \*
- D) Other (explain)

The first two hypotheses are spontaneously elaborated by students. These correspond respectively to a completely classical description of phenomena and a description coherent with a (local) hidden-variables interpretation. The third hypothesis, coherent with a QM interpretation, generally emerges from the recognition of the limitations of the first two in describing all the phenomenology (Michellini et al. 2001).

### 3.3 Interpretative hypotheses exploration

For each of the reported hypotheses, we propose a worksheet aimed at testing its validity by means of selected (ideal) experiments and at exploring consequences implicated. By means of close questions, the students make their predictions, based on the explored hypothesis, about the results of the proposed experimental tests. Afterwards they compare their predictions with the experimental results. Thus, the students are asked to draw a conclusion concerning the validity of the explored hypothesis. Finally specified situations are proposed in order to recognize the main consequences related to the considered hypothesis.

The worksheets ConE-F-G propose to test the hypothesis A. The goal of worksheet E is to recognize the physical distinction between a pure quantum state (a superposition of two orthogonal states) and a mixed quantum state (a statistical distribution of the previous orthogonal states). Students analyse the interaction of  $45^\circ$  polarized photons with vertical/horizontal polaroid. The worksheet F faces the knots and their interrelations: uncertainty, the uncertainty principle, the incompatibility of properties, the identity of equal quantum particles. These different aspects are required to be contextualized by students when considering the polarization phenomena.

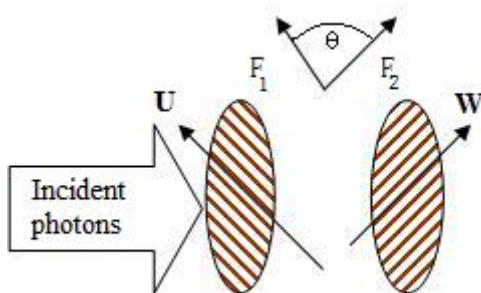
The worksheet G proposes the subtle knot of the impossibility to attribute a trajectory to a quantum system, one of the well known learning problems [2]. It is proposed to re-analyze, in ideal situations, the interaction of polarized photons with two inverse aligned birefringent crystals, explored in real situation with the worksheet FenC.

The worksheets ConG and ConH propose to test the hypothesis B, following a modality similar to the one in the hypothesis A, considering the two situations of polarized photons interaction with polaroids and birefringent crystals. In order to activate the cognitive crisis about hypothesis B in the students, it is crucial to consider, like in worksheet ConH, the case of photons transmitted by two aligned birefringent crystals, where the second is  $45^\circ$  rotated around the direction of the incident beam [25]. This situation is proposed in real context in the worksheet FenC.

The worksheet ConI proposes to analyse the hypothesis C, the QM one, that is activated by recognizing the active role of polaroid in the interaction with photons. We propose to re-analyze four situations on the basis of hypothesis C, where the beam of polarized photons strikes: A) a polaroid; B) two polaroids rotated at  $45^\circ$  one w.r.t. the other; C) a calcite crystal; D) two crystals of calcite, one direct and one inverse. For each analysed situations, students are requested: to recall the main experimental results, and to describe the processes involved in terms of Hypothesis C.

### 3.4 Worksheet Form A - Quantum states and vectors

The worksheet Form A aims at building the formalized representation of the quantum state, to recognize the role of linearity in the formal representation of the superposition principle, underlying the role of the scalar product in evaluating the transition probability, recognizing the non-classical nature of quantum interference which is the main manifestation of a superposition of states, addressing the connection between linear operators and physical observables of a quantum system, relating projectors of states and selective measurements. In the following we analyze in detail the seven steps (D1-D7).



**Figure 2** The photons, transmitted by polaroid F1, oriented according to the unit vector  $\mathbf{U}$ , are striking on a second Polaroid F2, oriented according the unit vector  $\mathbf{W}$  [ $(\mathbf{U} \cdot \mathbf{U}) = 1$  and  $(\mathbf{W} \cdot \mathbf{W}) = 1$ ].  $\theta = 0$  corresponds to a maximum of transmission.

**D1** - The student recognizes first the possibility to describe by means of a unit vector the orientation of a polaroid with respect to an arbitrary framework of reference. This unit vector is parallel to the surface of the polaroid and orthogonal with respect to the propagation direction of incident light. We suggest to analyze a typical experimental set-up to explore Malus law (Fig. 2) in order to recognize the connection between the probability of transmission  $P_t$  and the scalar product  $\mathbf{W} \cdot \mathbf{U}$  of the two unit vectors  $\mathbf{W}$  and  $\mathbf{U}$  individuating the orientations of two polaroids. The students are requested to face the following issues: what is the probability  $P_t$  that a photon transmitted by F1 is also transmitted by F2? How may we express  $P_t$  as a scalar product  $\mathbf{W} \cdot \mathbf{U}$ ?

**D2** - In the second step the students recognize the possibility to represent the state of photons before the measurement with vectors  $\mathbf{u} // \mathbf{U}$  and the state after the measurement with vectors  $\mathbf{w} // \mathbf{W}$ . With multi-choice questions the students face the following issues: a) The probability  $P_t$  of transmission of a photon that strikes F2 is completely defined by  $\mathbf{u}$ ? b) The state of the photon before striking F2 is completely defined when  $\mathbf{u}$  is defined? c) may we conclude that the statistic behaviour of a photon is completely defined when  $\mathbf{u}$  is defined?

To conclude this step the students are requested to clarify the following sentence, using the introduced formalism: The probability  $P_t$  represents the probability of transition between the two states of the photon.

**D3** - The formal representation of the superposition principle is proposed starting from the the probability of transition  $\mathbf{u} \rightarrow \mathbf{h}$  and  $\mathbf{u} \rightarrow \mathbf{v}$ , with  $\mathbf{h}$  and  $\mathbf{v}$  unit vectors mutually orthogonal (horizontal and vertical respectively). By means of sentences to be filled in, the connection between the scalar products  $\mathbf{u} \cdot \mathbf{h}$ ,  $\mathbf{u} \cdot \mathbf{v}$  and the (real in this context) coefficients of the linear combination:  $\mathbf{u} = \Psi_1 \mathbf{v} + \Psi_2 \mathbf{h}$  is made explicit. In a 'map' question the students relate  $(\Psi_1)^2$  and  $(\Psi_2)^2$  and the right probability of transmission. As control-check questions, students are asked to make explicit the meaning of the normalization condition  $(\Psi_1)^2 + (\Psi_2)^2 = 1$  and the re-evaluation of the two probabilities of transitions  $\mathbf{u} \rightarrow \mathbf{v}$ ,  $\mathbf{u} \rightarrow \mathbf{h}$ .

**D4** - By means of sentences to be filled in, students address their attention to special situations in which probabilities of transition are 1 or 0 (sure cases). In open questions, the student must associate the first case to the  $\mathbf{v} \rightarrow \mathbf{v}$  or  $\mathbf{h} \rightarrow \mathbf{h}$  transitions and the second case to the  $\mathbf{h} \rightarrow \mathbf{v}$  or  $\mathbf{v} \rightarrow \mathbf{h}$  transitions.

**D5** - Multi-choice questions and sentences to be filled in allow students to recognize the quantum interference, implicitly involved in the formalism and alien to the classical concept of probability.

**D6** - As a conclusion of this part of the worksheet the student briefly discusses the meaning of the superposition of quantum states, referring to the state of polarization at  $45^\circ$  as a superposition of the states  $\mathbf{h}$  and  $\mathbf{v}$ . They finally comment the physics meaning and the formal expression of the quantum superposition principle.

*D7* - In the previous points we introduced the vectorial representation of both quantum states and the principle of superposition, recognizing the role played by the scalar product in determining the probability of transition. Now we shall explain the connection between the scalar product and projection operators, which will lead us to the representation of physical observables with linear operators. The context is: a photon beam prepared in the state represented by the vector unit  $\mathbf{u}$  (ex.: photons transmitted through a polaroid, oriented according to the vector unit  $\mathbf{U}/|\mathbf{u}|$ ) strikes a (ideal) polaroid oriented according to the vector unit  $\mathbf{V}$ . The students complete the mathematical expression of the probability of transition. Once the possibility to represent  $P$  as

$$P = (\mathbf{u} \cdot \mathbf{v})^2 = (\mathbf{u} \cdot \mathbf{v})(\mathbf{v} \cdot \mathbf{u}) = \mathbf{u} \cdot (\mathbf{v} \mathbf{v} \cdot) \mathbf{u}$$

has been introduced, the students are requested to recognize the mathematical meaning of the mathematical object  $(\mathbf{v} \mathbf{v} \cdot)$ . A brief conclusion stresses the physical meaning of this operator.

In the worksheet Form B we construct a general linear operator. The conceptual path is the following. Evaluating the expectation value of a physical observable (the polarization), we assign a weight to each possible measurement result corresponding to the probability of transition: the square of the scalar product between the vector of the initial state of the system and vector in the corresponding final possible state (auto-vector). By means of the projector  $(\mathbf{v} \mathbf{v} \cdot)$ , it is simple to build the operator that represent the observable of a measurement. According to how this is made, its auto-vectors are the vectors of possible final states of that measured system and its auto-values are the corresponding possible results of the measurement. In this way, the connection between linear operators and physical observables is made explicit, as well as the physical meaning of this connection is understood.

#### 4 CONCLUDING REMARKS

The presented worksheets constitute an integral part of the didactic proposal for the teaching/learning of QM and is the outcome of previous researches. They have been developed for the in-class work of students and they reproduce the modular structure of the path, allowing different ways to use them and also different ways to choose a path by teachers which use them. They have been developed in order to offer students orientation of independent work, promoting the comparison of ideas in students groups, fixing ideas and concepts through making them explicit. They allow teachers to follow the learning path of students, identifying unsolved knots, recognizing those steps in which the learning actually took place. Thus, they also provide precious indications for researchers about how students approach QM concepts, and how those concepts are activated by operative contexts. They finally constitute a useful working tool in activities for teachers formation by means of experiential modalities.

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