TEACHING THE HEART OF QUANTUM THEORY

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1 HOW TO GET TO THE HEART OF QUANTUM THEORY

Most difficulties in understanding quantum theory have their roots in daily experience which relies on objects having definite properties and on the perception of things being separated from each other and their environment. These well known classical experiences then serve as a natural background for understanding quantum theory. Accordingly, it is often tried to explain the peculiarities of quantum theory e.g. in relation to the classical concepts position and momentum or to the duality of wave and particle. But this often results in misleading visualizations and misinterpretations.

1.1 Basic mathematical concepts and their interpretation

For teaching and didactical reconstruction it is important to identify the most crucial points for understanding. The experiments by Zeilinger and many others suggest that the properties of superposition and entanglement characterize the differences between quantum physics and classical physics. In the following I will present the relation between the most basic notions in quantum theory and its philosophical implications.

States and Superposition The use of "states" and the consequences of the principle of superposition are of fundamental influence for the behaviour of quantum objects. It is the basis for the appearance of the non-determinism of quantum theory. The linearity that shows up here is not found in classical theory. Of course there are linear approximations in classical physics, but an exact computation often would have to imply non-linear terms. In quantum theory, however, the state equation is linear in the state itself because of the quantization mechanism. This mathematical property has conceptual consequences: One may say that superposition means something like a logical (non-exclusive) "*or*" which simply does not occur in the classical description of nature. This focuses on the logical and mathematical incompatibility between quantum physics and classical physics. Nevertheless it opens a path to an intuitive understanding of the behaviour of quantum objects and an "everyday interpretation" of the superposition principle in terms of human behaviour. It gives an idea what is meant if we say that a quantum object does not possess definite values for its "abstract" properties before measurement.

Observables and Operators Physical quantities, the observables, are represented by operators defining rules of transformation. Hence operators on the one hand represent an active concept - by acting on state vectors - on the other hand they

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incorporate in their eigenvalues all possible outcomes of measurements. The most simple example for all these concepts are the spin operators whose action and eigenvalues can be calculated easily. With this example it can be shown how general states can be developed into eigenstates of an operator. As a consequence one and the same state has different representations with respect to the eigenvectors of different operators. Herewith the operators reflect the principle of superposition and at the same time the indeterminism: one never knows in advance which of the eigenvalues will be realized (with exception of pure states). However, even the property "pure state" may change from one operator to another, as for instance in case of the spin operators giving rise to the

Heisenberg Uncertainty Relation One of the points most often misunderstood is the uncertainty. The early interpretations of Heisenberg and Bohr suggested a viewpoint such as if the measurement would be inexact, but the core could be understood in a classical manner. The core of the notion, however, lies in the representation of physical quantities as linear operators and their non-commutativity. Non-commuting operators do not have common eigenstates. Consequently the corresponding observables cannot have fixed values simultaneously. In contrast to the Heisenberg uncertainty relation for position and momentum the spin allows for a concise and unique statement of non-existing fixed values for properties.

From a philosophical point of view the complementarity distinguishes an ontological vs epistemic interpretation of properties. Uncertainty forbids to say what a quantum object *is* or which properties it has in a certain moment.

Entanglement Perhaps the main characteristic of quantum theory compared to classical physics is the entanglement. It combines the superposition with uncertainty and relies completely on the mathematical apparatus: the construct of Hilbert space and the fact that systems of quantum objects have to be described in the tensor product of their own Hilbert spaces. It is interesting that the entanglement may be independent of space and time. This property has as a consequence the non-separability of quantum objects, which cannot be understood in the framework of classical notions. The entanglement is responsible for the non-separable effects observed in realizations of EPR-thought experiment, the transformation of quantum properties from micro-objects (as atoms) to macro-objects such as Schrödinger's cat and allows e.g. for the massive parallelism of quantum computers.

Measuring process There are many different interpretations of the mathematical formalism of quantum theory. Their main differences lie in the acceptance and understanding of measuring process. The core difficulty is the emergence of classical facts from the quantum-physical superpositions. One path to a possible interpretation may be the notion of information and information exchange between the quantum system and its (classical) environment. A substantial difficulty lies in the fact that the description of expectation values needs the (Euclidean) metric of a

Hilbert space which is nonlinear in contrast to the linear structure of quantum theory itself.

2 APPROACHES TO QUANTUM PHYSICS

Historically quantum physics has two roots: the black body radiation of thermodynamics and the atomic spectra. The thermodynamic approach requires deep insight into physics whereas the atomic spectra can be shown easily and may be explained on phenomenological level. Those led to the theoretical formulation of quantum physics by Heisenberg, the corresponding formalism and then to the Schrödinger equation. But soon interpretational problems arose, starting a century-long debate. Since about 1980 experiments with single ions, photons etc are being performed which provided new insight into the peculiarities of quantum physics and opened new ways of presenting and discussing the interpretations.

2.1 Classification of Approaches

Quantum physics has so many aspects that different approaches can be chosen. With each approach the interest of the teachers and their students has to be taken into account as well as their physical and mathematical abilities.

Normally, almost traditionally, the *historical approach* is chosen: It starts with the quantum character of photons as expressed in the Einstein-de Haas experiment, the analysis of atomic spectra and related experiments. Mostly it treats as a central point the duality of wave and matter.

If applications are of special interest the *technological approach* would be well suited. It may focus on the applications of quantum theory such as lasers, diodes, superconductivity or similar subjects.

Some students are attracted by the *philosophical aspects* of quantum theory. Then an approach leading to its discussion would be most appropriate. It would present the difficulties of interpretations and understanding of terms like superposition, uncertainty or entanglement and the implications on world view.

In the light of new developments a so-called *fundamental approach*, stressing the main concepts of quantum theory may seem most appropriate.

All these approaches can be underlined by suitable chosen experiments such as the double slit experiment including electron diffraction and its variants stressing e.g. the so called which way information aspect shedding some light onto the measuring process. Seldom one of these approaches will be adopted in a pure manner; mostly a combination of perhaps two of these approaches will be adopted.

2.2 Didactical Considerations

Recurrence to the mathematical framework makes talking about the described features a lot easier. But at school the mathematical tools are available only on a very limited level. Hence it has to be considered to which extent the concepts can be simplified. The characteristics should refer to the behaviour of objects:

- Experimental results can not be predicted. (Superposition)
- Quantum objects have no fixed properties. (Uncertainty)

- Quantum objects react to „distant“ related quantum objects. (Entanglement)
- Only by a measurement a quantum object will attain a fixed value of a property. (role of measurement)

One of the central features described above is the uncertainty: A first glance to the most well-known uncertainty relations concerning position and momentum reveals that these are mathematically intricate. Furthermore the mixing of uncertainty with disturbance of measurement (statistical interpretation) is very common. It seems more appropriate to choose the spin with only two discrete eigenvalues to visualize the uncertainty. The simple experiments described in the paper of Michelini below lead as directly as possible to the basic phenomenon of non-existence of fixed values for some properties of single quantum objects.

Teaching goals Besides knowing facts and laws learning about nature of science is one of the goals of physics education. In the worksheets presented by Stefanel et al below, physical thinking is being promoted and leads to insight into the complex interrelation of theory and observation. Because quantum theory is open for different interpretations students learn the role of experiment as a "judge" on nature. The connection to philosophy shows the relevance of interpretation for the world view.

Teaching difficulties Classical terms are not adequate for quantum processes. This difficulty becomes obvious in the double slit experiment or a Mach-Zehnder-Interferometer. To describe the behaviour of the photons the wording “ways” or “trajectories” seems inadequate because it is quite close to classical associations. It could be of advantage to use the auxiliary term “possibilities”. Similar precautions have to be applied in the case of electrons in an atom. The image of the Bohr model, quite familiar to all, suggests the existence of trajectories. Here it could be helpful to go the path Heisenberg did in finding quantum theory: What can we know about an electron in an atom? The first information stems from the spectral lines of the atoms. They only give information on the energy but not on position and momentum of electrons. Hence all statements only may refer to those properties.

2.3 Proposal for approach

Especially the concepts of uncertainty and entanglement gained great importance in the last years. These concepts seem so fundamental that they should be conveyed at school. Concentration on these essential aspects seems to open up teaching paths that equally lead to an understanding of the most important traits of quantum theory as well as to its relevance to world view. The *fundamental approach* uses the simplest possible systems: two-state quantum objects. The visualization mainly relies on the polarization properties of light. There are basic experiments with calcite crystals and polaroids that can be done (see contribution of Michelini) These activities can be supplemented by aid of carefully constructed worksheets guiding the students (see contribution of Stefanel et al.).

An additional central notion is "information". The implications are demon strated in an experiment relying on the classical double slit experiment and visualizing the discussion on the so-called which way information with aid of different polarization at the two slits. Also a so-called quantum eraser can be built with simple materials. The discussed version of the double slit experiment allows for building on the pupils' pre-knowledge from optics. Furthermore the experimental device can easily be built by the pupils themselves. It is reported on results of experimentation activities and learning success in classroom.

3 CONCLUSION

Some remarks about the relation between the mathematics, the physics and their interpretation and visualization are in order now. Physics and mathematics seem to be very formal and strictly deductive: In physics hypotheses are made, there from well-defined experiments are conducted and the conclusion follows; in mathematics first there are the presuppositions, then the proof is given and the assertion follows. Only seldom the process with often intuitive considerations is described, perhaps because they are regarded as not sufficiently strictly founded even if visual ideas or analogies may play an important role in deriving the results. Often only in the second step the means for verifying or proving the conjectures are found. The above examples should indicate several different aspects:

- The mathematical structure of physics needs interpretation.
- It is impossible to visualize quantum physics adequately by analogies or models from classical physics by principle.
- Quantum physics leads to a world view different from the "clockwork universe" of classical physics.

All these aspects are important to the students in order to gain insight into physics beyond its formal front end. Furthermore, students should be able to recognize the borderline between esoteric interpretation and a scientifically justified interpretation.