ON UNDERSTANDING, EXPLAINING AND MATHEMATICAL FORMULATION OF PHYSICAL PROBLEMS IN SECONDARY SCHOOL

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1 INTRODUCTION

Physics as a natural science relying on experiences has observation and experiments as a fundamental source for gaining knowledge. However, the path from experiment to a general law or even theory is by no means straightforward. Every experiment needs hypotheses to be set up and a framework in which it can be interpreted. In this regard mathematics with its tools, theorems and theories is as important for physics as a whole as experiments. In order to be able to fruitfully apply mathematics to its problems physics has to discard some aspects of nature and to analyse it from specific view points - using the technique of idealisation and its own semantics. Hence the physical way of looking at nature requires a certain amount of formalisation, objectivation and mathematisation; it uses specific methods in the analysis of phenomena and processes. The body of knowledge and a repertoire of methods has been developed in a dynamical historical process still being in progress. This basic methodology has led to many important applications and technological devices shaping our everyday life.

2 GOALS OF PHYSICS EDUCATION

The basic goal of physics education is a stable broad knowledge of physics contents, comprising fundamental terms or notions like current or energy, concepts like the force concept, physics laws that describe relations between different quantities, and explanatory models as e.g. the particle model. But in the light of the previous statements the students in addition should have insight into the way physics as a science is conducted and how it arrives at its results. Its objectivity and rationality is to be conveyed and related to the necessity of interpreting experimental results. All these competencies should enable the students to participate in a rational, scientific discussion, communicate their knowledge about physics and gain judgements based on scientific insights.

These aims have to be achieved by means of concrete tasks in physics lessons in order that the students develop their competence and physics knowledge. Starting with own experiences students should understand everyday phenomena on a scientific level as well as the concrete meaning of abstract physics concepts and laws. Furthermore they should gain insight into the basic structure of physics. The overall aim is that they have the knowledge and ability of applying physics methods to new tasks. This implies not only experimental but also theoretical methods including mathematical ones such as the use of formulae and equations.

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Furthermore they should be able to communicate about physics contents, explaining them e.g. to peers or to other persons.

3 COMPETENCE IN PHYSICS

From these remarks it turns out the the trias of *Understanding – Explaining – Applying* plays an essential role in physics education. This will be elaborated in more detail now.

3.1 Aspects of understanding physics

The central point of physics education is that the students reach „understanding“. However, this term is not well determined and difficult to define. Understanding has several features - qualitative and conceptual as well as methodological (experimental and formal) understanding - which can be reached on different levels.

Levels of understanding In doing physics two aspects - theoretical thinking and concrete practical performing - are equally important and yet have to be differentiated: the difference of knowing „why to do“ *vs.* knowing „how to do“ has to be taken into account. From both aspects the feeling of „being used to“ can provide a surrogate or an illusion of understanding that allows for handling certain routines, perhaps by rote, mostly connected to a fixed context perhaps well known from everyday life.

1. Most basically practical experience or - on a quantitative basis - obvious „if...then“ - relations can superficially be applied to solve already known problems.
2. On the next level understanding would imply to be able to perform a deduction (on the theoretical side) or to know how to perform a known experiment by oneself (on the practical side) in only partly known situations.
3. A third level of understanding is reached if the students are able to apply a law or a model or an experimental technique to different unknown situations or contexts.
4. A very important fourth level is reached if the students can combine their theoretical with the practical knowledge, that is to use different representations of their knowledge.

Observing understanding However, often it is noticed that a good performance at standard tests need not imply proper understanding. Solving a given problem might only require the use of a correct formula without exactly knowing what lies behind. Depending on the concrete problem a calculation might be easier for the students than to explain a process qualitatively, where they have to choose the appropriate model or law and describe the relations between different physical quantities. At school it is an important task for the teacher to have some hints whether her students really understood the subject matter. This leads to the question: *„Can „Understanding“ be observed?“* In my opinion in general this will not be possible, but students can show their level of understanding with several tasks. With respect to experimenting the corresponding levels can be:

1. describing experiments;
2. explaining experiments or evaluating experiments - be from a numerical aspect or from the aspect of applying explanatory models;
3. varying experiments systematically after determining relevant parameters;
4. planning experiments on the basis of theoretical considerations, corresponding to the forth level above.

Furthermore the translation of a physical process into mathematical language in the sense of mathematical modeling often requires an explanatory step first. Therefore this might be a hint that the ability of translating physics to the mathematical language – again switching between different representations - could indicate „understanding". Hence, writing about physics processes – possibly also using equations or formulae – could be an appropriate way of deciding whether the students understood the physics content [1].

3.2 Role of Explaining

The term „explaining" plays a central role in physics lessons: the teacher explains to her students or the students themselves explain what they have conceived from the lessons. In a narrow sense explaining a physical process means to find its reasons in fundamental models or theories. Another aspect of explaining consists in clarification of fundamental concepts or interpretation of physics laws. Dependent on the listener (or reader) the use of appropriate models and of appropriate language is central.

Explaining takes place in the realm of language, it is verbal communication. But words can have different meanings depending on the context in which they are used – in everyday situations or in a social or scientific setting. Therefore the meaning of the words may be unclear to the learner. So in certain contexts some students may find mathematics a help.

3.3 Applying Physics

Applying physics in the same moment requires understanding and promotes understanding in kind of a hermeneutic circle. To successfully initiate such a process it is important to choose a suitable context which is apted to display the essential features of the physics subject in question. In addition the chosen context should enhance the comparison between theory and reality and often will require some calculation in order to get relevant results.

4 ROLE OF MATHEMATICS IN PHYSICS EDUCATION

4.1 Mathematical tools in physics

Doing physics without any mathematics is nearly impossible. Mathematical elements are necessary in many physics related activities:

1. *evaluating experiments:* drawing diagrams, computing quantities from raw data;

2. *deriving relations between physical quantities:* rearranging terms, evaluating functions;
3. *modeling a physical process:* identifying relevant quantities, applying conservation laws, mathematical modelling;
4. *structuring potential:* recognizing similarities between several subjects.

To design an appropriate teaching environment with respect to the use of mathematical tools in physics lessons the presuppositions from mathematics have to be analysed:

- Students know numbers, even if without units. The connection with physics consists e.g. in knowing typical everyday values for important quantities such as energy or force.
- Students know how to handle fractions or terms which is very important in physics in order to derive new insights e.g. the dependence between physical quantities.
- Students manage equations or use functions. Herewith, two different meanings have to be distinguished: an equation could either give the definition of a derived quantity or represent a function describing dependencies between several physical quantities.
- Graphical representations are used to show functional dependence (e.g. proportionalities), which has to be complemented by interpreting the functional dependencies from a physics point of view. An aspect reaching beyond the scope of school is the application of mathematical theorems for deriving physical structures, (e.g. Noether theorems).

4.2 Views of students

The students themselves have an ambiguous view onto mathematics in physics, [2]. Some seem to like it perhaps because the semantics of verbal explanations tend to inherit more arbitrariness from language than the apparent uniqueness of mathematical formulae. But on the other hand many students do not seem to like mathematics in physics lessons in spite of the clarity involved in using mathematics. This may have several reasons. There are additional difficulties compared to "pure" mathematics, some appearing trivial at a first glance but being quite intricate for the learners: Symbols such as p or E can have different meanings depending on the context. Formulae or equations play an ambiguous role: They allow for a compact representation of complex relations, but require a translation for the concretisation of physical relations. In the light of the ambiguous attitude it is quite important to find ways to show the students the role of mathematics and make its use – with stimulating questions - necessary and motivating.

4.3 Cognitive analysis of physics problems

Most curricula require that students should not only reach an appropriate understanding of how physics is done and what is the nature of physics or science in general, but also to apply their knowledge to concrete problems. Here we concentrate on tasks involving mathematisation and some kind of mathematical modeling. The hypothesis is that the abilities of understanding, verbal explanations and mathematical formulation are interrelated, [5]. We start with the relation between experimenting and the mathematical description which has much in common with mathematical modeling, [3]. Often the experiments in physics lessons are done quite carefully but the step from phenomenon to the mathematical formulation is gone too fast, [4]. Since the corresponding steps are so important they have to be analysed in detail.

Example of cognitive analysis For establishing a relationship between the experimental and the mathematical representation of a physical law the students have to identify significant parameters and the quantities they can measure experimentally and to connect them in the light of a theory which also serves as an interpretational frame. Very similar steps have to be pursued in the modeling procedure. We will do this with the example: „*Cooling by Evaporation*": a bottle with a soft drink should be cooled by wrapping it in a humid cloth.

- *Describing*: In a first step the physical phenomenon simply has to be described: The cloth first is wet, then gets dry, connected with cooling the drink.
- *Explaining*: This leads to recognizing the physics behind and the relevant physical laws: The water in the cloth is evaporating which requires energy taken from the surroundings. In this important step the core of understanding takes place.
- *Establishing the physical model:* Then the students have to analyse the process in more detail by identifying the relevant quantities and revealing their dependencies and relations: evaporation heat and the law of energy conservation and energy exchange. Up to now the students only move in the space between the real phenomenon and the physical model with no mathematical modelling involved.
- *Mathematical modeling:* Variables and parameters have to be identified and assigned to the process: the mass of evaporating water, specific evaporation heat, mass and material of the bottle and mass of the drink and their specific heats.

The further procedure depends on the goal: If the process – the change of temperature - in its exact time dependence for every minute should be described then more mathematics or programming skills (at least difference equations) are required, therefore suitable above grade 10, [6]. If only the final temperature ought to be determined provided the process runs sufficiently fast only a simple balance equation is needed and can be treated earlier in school, perhaps beginning from grade 8.

Difficulties in modelling It seems crucial that only quite late in the analysis the mathematical model is derived and the mathematical language is used. An important point is that the derived equation has to be interpreted and evaluated. Furthermore it has to be reviewed how the solution was found. The prediction can be tested in a home experiment, at the same time training the methodology of experimentation.

The modeling procedure can be difficult because it requires specific idealisation: e.g. sometimes friction has to be neglected, sometimes it plays the essential role in the process. For the students being mainly novices this is difficult to decide. Furthermore, sometimes the analysis of dimension (units) or knowledge from other parts of physics or experiments have to be invoked. All these steps require knowledge of mathematical symbols and operations and abilities in interpretation and evaluation of models. Therefore the ability to perform all these steps has carefully to be developed.

4.4 Choice of appropriate curriculum

In the light of the skeptical view of most students on mathematisation, the basis for motivation will be to rise curiosity, may be by surprising effects and to lead the students to pose questions where a numerical answer and hence a computation and mathematical modeling is necessary. Examples with respect to everyday life are of special relevance. The results of modeling should be tested by real experiments in suitable cases such that the students can control their considerations and their models. Then they should see the success of their modeling and gain self confidence.

First steps towards modeling Suitable examples for the beginning (grade six to eight) should concentrate on estimation of typical values. The students should develop the courage to use own values for physical quantities – e.g. force, power, energy or similar – and learn how to work with tables where they can look up special kinds of values, e.g. specific conductivity etc.. This ability might help them to deal with Fermi problems.

The next level of competence will be to use known physical laws in order to get interesting answers. In connection with definition of pressure the „Magdeburg Hemispheres“ invented by Otto von Guericke could be treated: Estimate the sufficient low-pressure inside the ball necessary to let 8 horses pull the halfs without breaking it! Modeling some sports - jump or ball play - could be instructive subjects beacuse they show the technique of idealisation such as neglection of friction, center of mass, etc..

Modeling complex procesess The next step is the analysis of dynamical processes. In order to being able to cope with more complex systems the idea of equilibrium should be introduced. It allows to concentrate on balancing equations which relate start point to end point and give insight into the power of conservation principles, especially energy conservation, which play a central part in physics. The example of "cooling by evaporation" has been treated in detail. Further problems are:

- Determine the equilibrium temperature in a greenhouse!
- Evaluate the influence of friction in free fall.

Fascinating examples could be taken from interdisciplinary problems. Complex interrelations must be analysed and important parameters be identified. From the viewpoint of situated learning or contextual settings there one could rise questions e.g. concerning the human biology in its environment:

- „Why is one more cold in water than in air? “ (using heat transport)

- "How much food needs a cyclist during a race?" (energy balance and efficiency)

Dynamical modeling In the higher grades (11 or 12), however, one would like to have more insight into the dynamical process. Then difference equations have to be used whose solution needs some programming technique or use of modeling software with appropriate systems allowing for graphical „programming" (STELLA, PAKMA or similar products), where the numerical part is hidden in the symbols and which allow for intuitive „equation solving". An additional viewpoint is the importance of physical considerations for society enabling the students to participate in the social discourse. Recent subjects are e.g. the production of biological fuel or the effect of gulf stream in the framework of global warming. These problems need well founded estimations on a physical basis. Here the students need own experiences with simple modeling in order to understand what is meant by e.g. climate modeling and to be able to judge related statements.

5 CONCLUSION

In a physics course there has to be a balance between different aspects of doing physics: observation, explanation, structuring processes and mathematisation which is necessary for getting definite results. All these aspects are necessary to give the students insight into the structure of physics and opening up new motivating contexts for treating physical problems. It is under way to develop and evaluate suitable modeling and mathematization tasks in a curriculum for reaching these goals.

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