Modeling in the Classroom: Linking Physics to Other Disciplines and to Real-life Phenomena

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Abstract
New technologies provide us with powerful instruments for the modeling of natural phenomena, thus increasing the variety of situations which can be examined in an introductory physics course. At a general methodological level, the combined use of modeling and data acquisition systems allows teachers to highlight one of the most fundamental aspects of scientific activity, viz. the relationship between theory and experiment.
By focusing on the structure which underlies the functional relationships involved in a given situation, modeling also fosters the acquisition of transversal and interdisciplinary skills. The cognitive acts carried out by the students are particularly interesting because – and this is true above all for work-environments which offer a graphic interface – the students are required to consider not just singular, isolated notions, but rather a whole network of connections, a veritable conceptual map informed by quantitative aspects.

Preliminary considerations

In this paper I shall present some reflections that arise from my direct experience introducing high school students, and to a lesser extend pre-service and in-service teachers, to modeling activities. As the title suggests, I shall limit myself not just to modeling in physics, but I also want to look at some aspects of modeling activity that can contribute to a better and deeper understanding of scientific activity as a creative but rigorous and coherent design for co-coordinated science teaching.
Since it is possible that not all high school teachers have direct experience with modeling at this level, I have structured my presentation with the following questions in mind: Is it really possible to introduce high school students to modeling? What can they learn? What is the best way to proceed? And perhaps the most intriguing question: What is the advantage of such a didactical approach? My aim is not to convince you that my personal answers are correct, but rather to present some elements which may stimulate reflection on this matter.
Examples and materials used in the present paper arise from my practical work with high school students or with pre- and in-service teachers I have introduce to modeling activities[2]. Obviously the skills of these two groups

[2] They are my own high school students from 16 to 18 years of age at the Liceo cantonale di Locarno, in total about 80 students during the last six years in groups of 15 to 20; the pre-service and in-service teachers come from different post-secondary institutions such as the Alta Scuola Pedagogica of Locarno, the SSIS at the Università di Udine and at the Università di Padova.

and the methodology required to teach them are completely different, but it is interesting to compare the reactions, the questions and the solutions proposed by high school students on the one hand, and graduate students on the other.
A further remark concerns the high school students' knowledge of physics at the moment I introduce the modeling activities[3]: they are used to seeing natural processes on the basis of a model that stresses the role of the *extensive physical quantities,* that can be stored in a system and that can be transferred from one system to another by virtue of a "driving force", i.e. through the action of a *potential difference*. In particular the high school students are introduced to thinking in terms of the different extensive and intensive quantities that characterize the different fields: in hydraulics water volume, pressure difference and water current, in electricity, electric charge, electrical potential difference and electrical current, and in mechanics, momentum, velocity difference and force. Furthermore, students can already make use of the so-called *balance law*, i.e. the equation that in different contexts expresses the relationship between the instantaneous rate of change of the stored extensive quantity, the intensity of the flows through the boundaries of the system and, if at all admissible, the rate of production/destruction of the quantity in question.
As far as the mathematical background of the students is concerned, they have already been confronted with the basic ideas of a numerical algorithm and of an iterative procedure. In particular they can apply the Euler method to determine time evolution for a specific quantity by a given process: they have worked out laboriously on paper some iterative loops for suitable examples (that do not necessarily involve physical quantities, or where the results are easy to predict or already known from previous school activity) and they have also performed the same procedure in a more automatized manner, for example by using an electronic spreadsheet.

1. Introducing modeling to high school students

To introduce my students to a modeling tool, I have chosen software that allows them to work on a graphical surface. Physical quantities are introduced by one of the graphical symbols, four in all, while the relations between them must be implemented explicitly. If we consider a mechanical situation, for example a rigid body with two forces acting on it, we have the situation represented in Figure 1.

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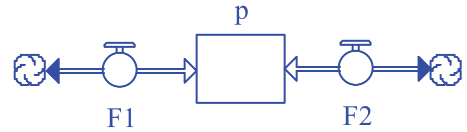
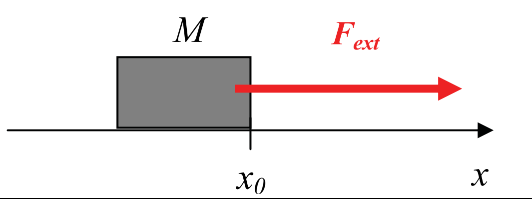
[3] For a general presentation of this approach see F. Herrmann *The Karlsruhe Physics Course* [1]; for the English version of the course for high school students see www.physikdidaktik.uni-karlsruhe.de.

Figure 1 - Basic elements: a rectangle represents quantities stored in a system at a given moment while the arrows indicate that there is an exchange between the system and its surroundings.	

We can read the diagram as follows: the rectangle represents the momentum of the rigid body; the initial value must be explicitly specified. The arrows represent the instantaneous momentum-flows between the system and the surroundings, while the small cloud symbolizes the surroundings. The whole scheme represents the *law of balance* for the momentum of the body: in fact, by drawing this combination of symbols we have implemented a first order differential equation.
As a first concrete example I often present the following situation: a rigid body (of given inertial mass *M*) which can move without friction on a horizontal surface and which is acted upon by a constant force (Figure 2).



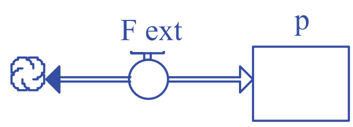


Figure 2 - A constant force acting on a rigid body: visual and schematic representation.

Focusing on the law of balance of momentum, i.e. on the dynamical aspects of the situation, the students will be able to write down, or rather draw, the following Newtonian equation

$$(1)\ \dot{p} = F_{ext}$$

For the students this means, expressed in words: "The force acting on a body determines how fast the momentum changes." They also understand that, for an interval of time where the force is constant, this can be expressed by

$$(2)\ p(t) = p(t - \Delta t) + (F_{ext}) \cdot \Delta t$$

This is the form we commonly use to write our *laws of balance*. It is identical to the simplest numerical method used by the software to solve the differential equations iteratively. As already pointed out, before using the system dynamics tool, I have my students perform such a solution procedure on paper, then with the help of a spreadsheet.

The resulting equations that represent the model of our example can be visualized at will simply by activating the corresponding mathematical or formal level in the software:

$$(3)\ p(t) = p(t - dt) + (Fext).\ dt$$
$$INIT\ p = 0\ \{N.s\}$$
$$INFLOWS:$$
$$Fest = 2\ \{N\}$$

In order to create more complex models we require two further elements: auxiliary quantities and connectors (Figure 3).

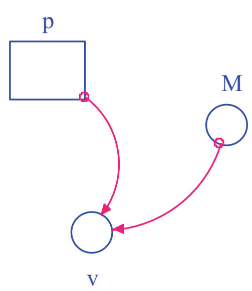


Figure 3 - Auxiliary quantities and connectors: these two symbols complete the tools we need in order to construct more complex models. The auxiliary quantities can represent a constant quantity, or a quantity that depends on other quantities, or it is also possible to relate measured data. The connectors indicate the interdependence between different quantities: the mathematical relationship must be inserted by hand.

The *auxiliary quantities* are represented by a circle and can be defined in different ways: for example as a constant quantity, simply indicating a numerical value, as here for the mass of the body; or alternatively as a function of other quantities, such as velocity in this example, given by the Newtonian constitutive relation "Velocity equals momentum divided by inertia". The last symbol, the thin arrow, is the so-called *connector*, which indicates the interdependence between the different quantities. In our example the velocity of the body depends both on its momentum and on its inertia. At this stage, the kinematical aspects can be introduced: the velocity of the body is interpreted as the instantaneous rate of change of position, so that the latter can be obtained by a stock-flow diagram (v. Fig 4) that, from the mathematical point of view, functions as an integrator.

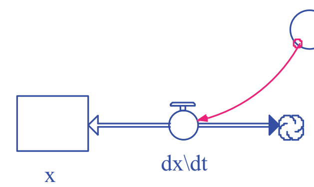
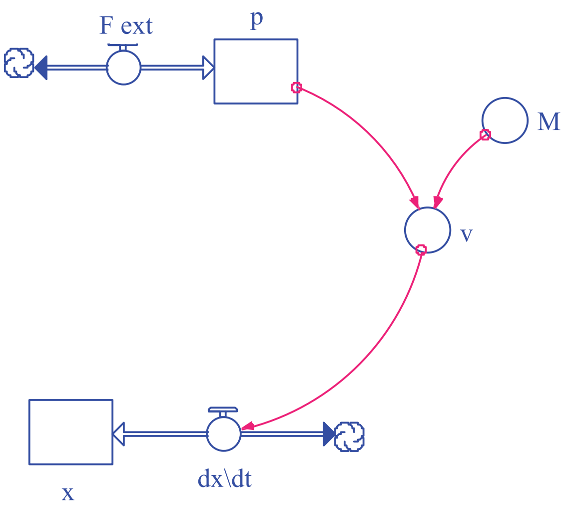


Figure 4 - Kinematical aspect: the actual position is obtained adding the displacement to the initial position.

To recapitulate, starting from a given problem, we have *constructed*, step by step, a possible solution: we can now consider the model as a whole (Fig 5)



$p(t) = p(t - dt) + (Fext) * dt$
$INIT\ p = 0$
INFLOWS:
$Fext = 2\ \{N\}$

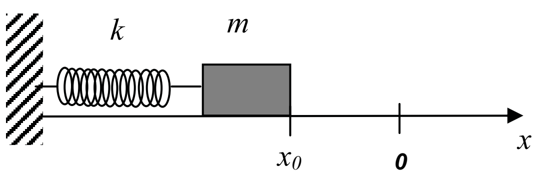
$x(t) = x(t - dt) + (dx/dt) * dt$
$INIT\ x = 0$
INFLOWS:
$dx/dt = v$

$M = 4\ \{kg\}$
$v = p/M$

Figure 5 - The complete model: beside the graphical surface, the mathematical formal level is easy to access: there the graphically implemented general relationships are visible as algebraic algorithms, as well as the particular laws that define the model. The results can be visualized in the form of a graph or of a table.

Starting from this first example it is interesting to challenge the students with a second mechanical situation, namely, the *harmonic oscillator* (Figure 6).

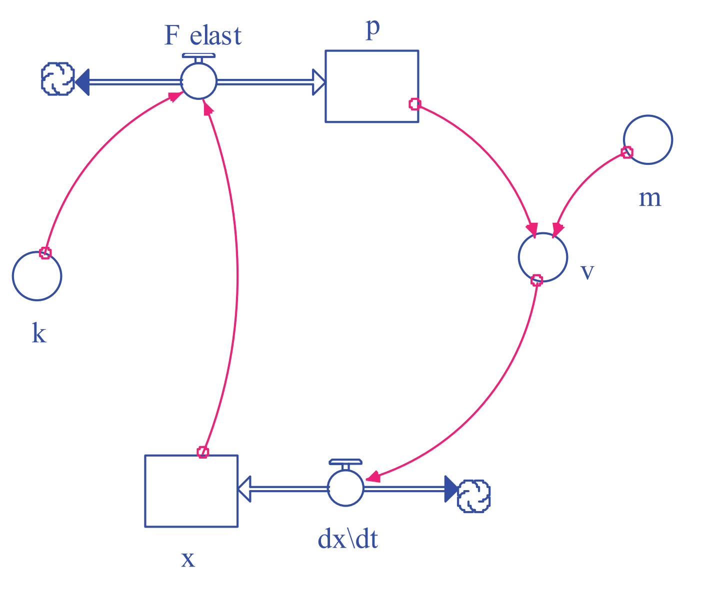
Figure 6 – The harmonic oscillator: an object of inertial mass m=0.4 kg is attached to one end of a spring with elastic constant k=26 N/m fixed to a wall. The object is shifted from the equilibrium position ($x_o = -0.08$ m) and then released.



We have here what at first may seem to be a completely different physical situation, but … let us examine the previous model more closely, and ask ourselves which parts of the model are still useful, or alternatively which parts of the model must be changed.

Let us consider the model again in detail: the law of balance of momentum is still valid; the same holds for the constitutive law for velocity expressed as a ratio between momentum and inertia, the velocity of the body interpreted as the rate of change of position, and the same for the kinematical relation with the position. What, then, should be changed? In this way we see that in this new situation there is only one new feature: the constant force should be replaced by an elastic force to account for the action of the spring (which we will consider an ideal spring). We must introduce therefore into the model an elastic constant (k) to account for the elastic properties of the spring and implement Hooke's law in the model. Finally, the initial conditions must also be up-dated (Figure 7).

At this stage students generally expect that this new model will work very well, and they start the simulation. The result brings deep disappointment: instead of the expected oscillation it shows a huge increase of the relevant physical quantities. What is wrong with the model? With students this is an interesting point, because in this way they experience directly that a numerical tool is not a magic instrument and that there are still some limits to its applicability.



*p(t) = p(t - dt) + (F elast) * dt*
INIT p = 0
INFLOWS:
*F elast = -k*x*

*x(t) = x(t - dt) + (dx\dt) * dt*
INIT x = -0.08 {m}
INFLOWS:
dx\dt = v

k = 26 {N/m}
m = 0.4 {kg}
v = p/m

Figure 7 - The model for the harmonic oscillator: the only difference with the previous model is the introduction of the elastic constant k and the implementation of Hooke's law.

Students must first pay attention to the choice of the iteration interval; up to now the default settings of the tool have not been questioned: in the present situation, the default choice (0.25 seconds) is not appropriate because it is of the same magnitude as the expected oscillation period. So students can try with a much shorter iteration interval, for example 100 times smaller. The result is much better in that the solution becomes oscillatory, but it is still not satisfactory, on account of the continuous increase of the amplitude.

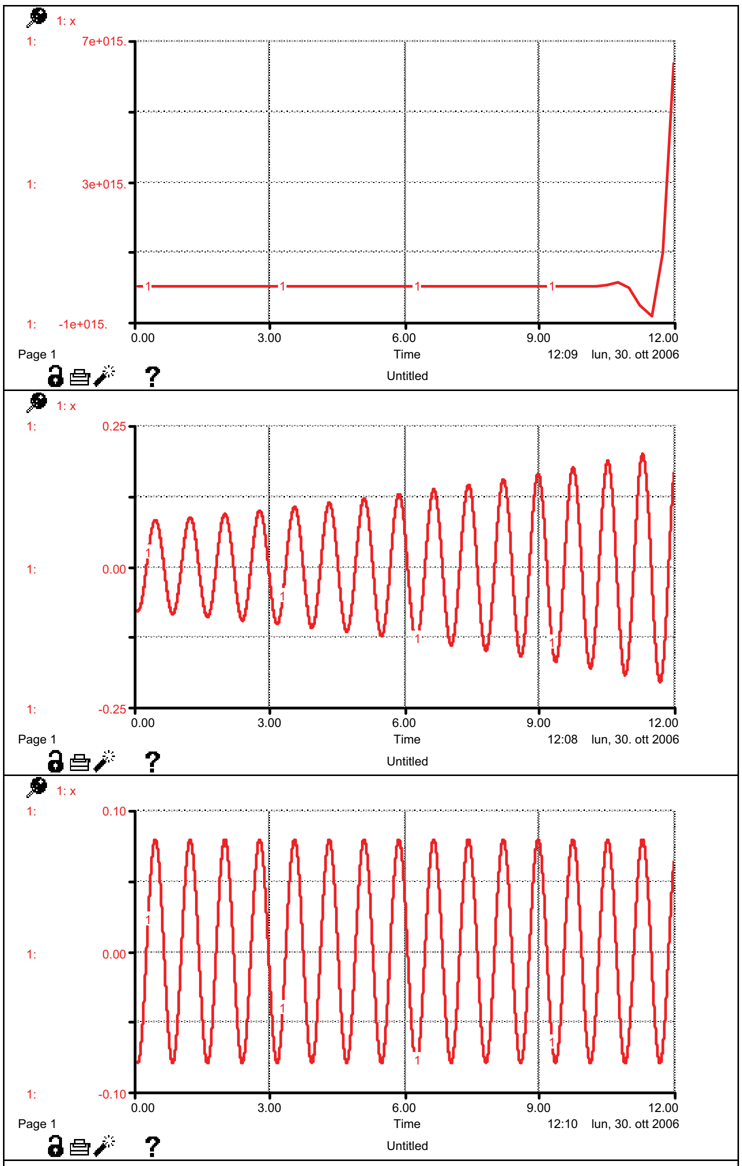


Figure 8 – Numerical aspects: students gain the feeling that numerical methods always have limits of applicability.

This second point is more difficult to discuss explicitly with students, because it is connected with the peculiarities of the integration method used. In fact, here, one comes up against the limits of Euler's method: by a more refined method (for example Runge-Kutta 4 method) students finally obtain a satisfactory result. At our level a closer investigation of numerical methods is not considered necessary: the most important point is that students gain operatively the feeling that numerical methods always have some limits of applicability.

The next step could be the introduction of viscous friction acting on the body. To complete the previous model, it is sufficient to add a new force, i.e. an additional momentum flow (Figure 9a). For the viscous friction force we assume the standard form

(4) $F_{friction} = -b \cdot v(t)$

where b is a coefficient whose value must be specified or, rather, determined from a comparison of simulation and experimental results previously obtained by the students. The model results show the expected damping effect (Figure 9b).

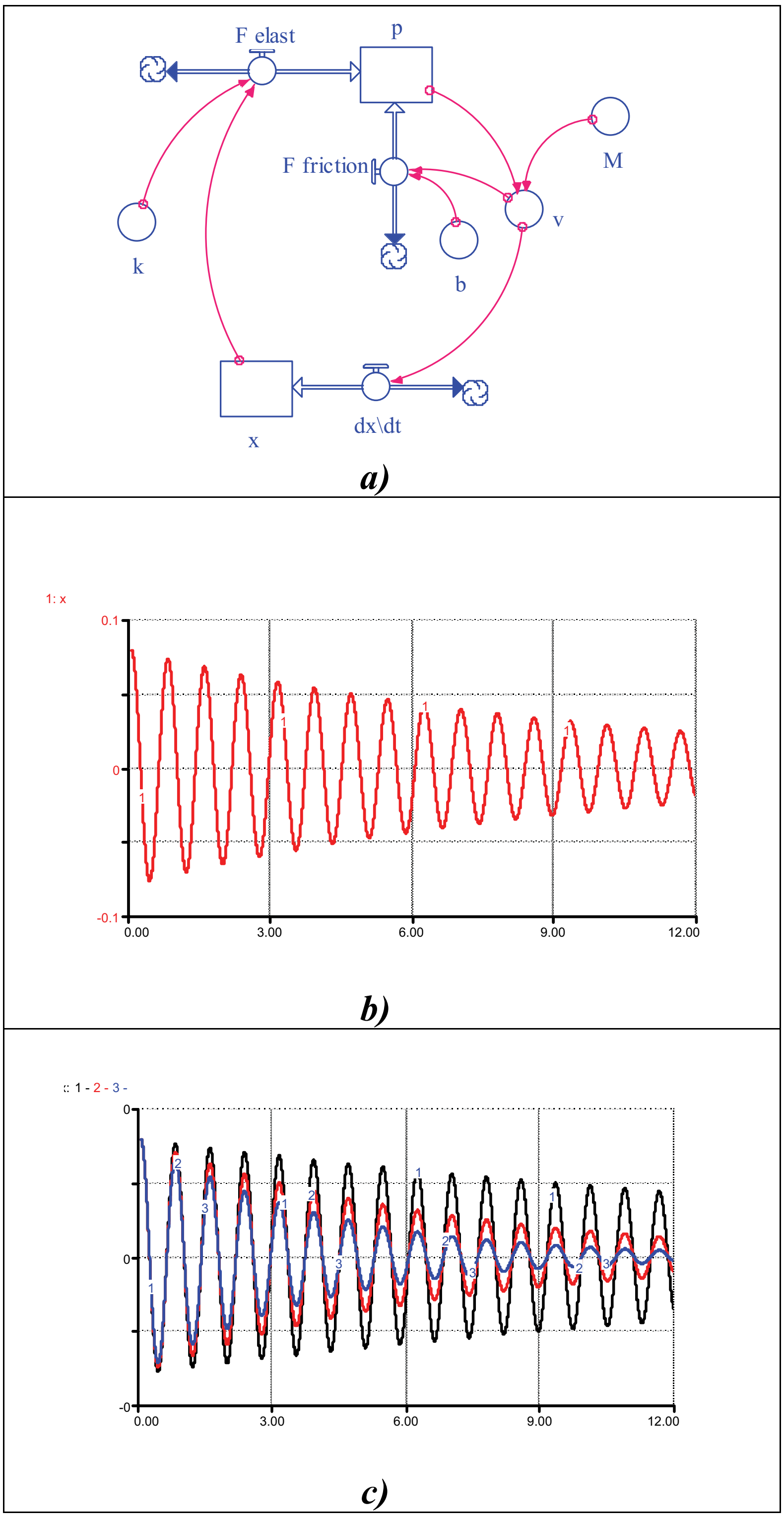


Figure 9 - Damped harmonic oscillator:

a) the model is completed with the (viscous) friction force;

b) as a result, the amplitude of the oscillation decreases with time;

c) to explore the effect of the damping coefficient b, students can start a batch of runs with different values of b.

An interesting feature is the possibility to have a batch run for different values of a given parameter (Figure 9c): students can compare the predictions of the model for various values of the viscous coefficient b. They appreciate this feature, because they can explore the induced effects and in this way they take a first step toward the understanding of the physical meaning of this physical quantity.

2. Co-ordination with biology and chemistry

My next point is about the role that modeling might assume for the co-ordination in science teaching at the high school level (i.e. biology, chemistry and physics). First of all, one can ask why advocate a *coordinated approach* to science teaching? The answer to this question can be summarized as follows: students should have the possibility to appreciate science as a vast, coherent and comprehensible description of natural phenomena. Science teaching therefore must be planned in such a way that students can, in fact, recognize this unity.

Of course biology, chemistry and physics each have their own special aspects: these must be acknowledged, maintained and highlighted. But they also share a core of knowledge, a sort of *transversal conceptual frame* that is common to all scientific fields. I believe that for the students it is of primary importance to recognize these features. The learner must have the possibility to construct step by step a coherent image of natural phenomena[4].

The main question in our present context is to point out what system dynamics modeling can contribute to this issue. System dynamics modeling allows all three sciences to make explicit use of quantitative methods, to construct dynamical models, and to compare models to reality. System dynamics modeling, which highlights the structure of relationships, also favors the use of analogical reasoning; this can be substantially enhanced adopting the "extensive / intensive" model that we discussed previously, in particular introducing *from the beginning* the pairs entropy / temperature in thermodynamics, amount of substance / chemical potential for the transformations of matter, in such a manner that the conceptual reference frame at the disciplinary level is then completed as shown in the following table:

	Extensive quantity	**Intensive quantity**
Hydraulics	Water volume	Pressure
Electricity	Electric charge	Electrical potential
Mechanics (translations)	Momentum	Velocity
Thermal processes	Entropy	Temperature
Chemistry	Amount of substance	Chemical potential

Table 1 - The conceptual reference frame at the disciplinary level.

With these tools available, there is a virtually unlimited choice of examples, such as chemical reactions and equilibrium, perturbation of the chemical equilibrium, chemical reactions and energy balances, electrochemistry (Nernst law, Daniell-cell, hydrogen-cell, …), pH, titration, extraction, osmotic pressure, cell membrane permeability (red

[4] For more details see the contribution A titration experiment as an example for a co-ordinated approach in science teaching at high school level presented at the 2005 Girep Seminar in Ljubljana [2] in which a co-ordinated approach based on the introduction of cognitive organizers is discussed.

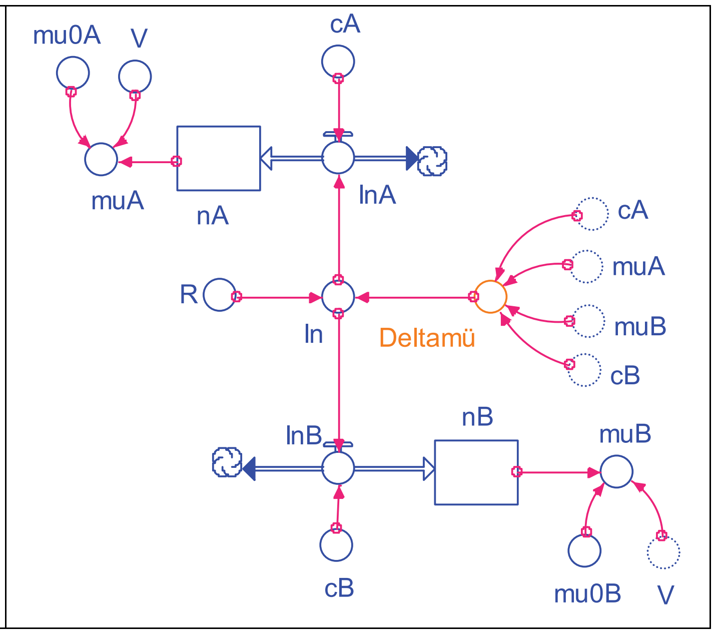
condition is to work with colleagues who share this approach and who directly or indirectly can refer in their didactical practice to this model. Students appreciate the coherence of the whole and they feel involved in the conceptual construction. As an illustration, let us consider a (hypothetical) reaction where the substances *A* and *B* can change into another under the stoichiometric conditions $3\,A \leftrightarrow\ 2\,B$ and ask, given certain initial conditions, what happens in the system.
Here we have an example that stresses the use of analogical reasoning. The idea is to use student's previous knowledge in hydraulics: as shown in the following scheme (Figure 10), the volume of water corresponds to the amount of substances, while the pressure difference corresponds to the chemical potential difference. The reaction will start spontaneously in a certain direction, in such a way that the chemical "driving force" decreases, until it vanishes. In this situation the system has reached equilibrium. Figure 11 shows the model that sums up the previous ideas. In a first situation, the initial conditions are chosen so that at the beginning we have a lot of A and very little of B. Can we predict what will happen in the system? And what would happen if the initial values were different? Say, a lot of B, and only a little of A? The model will allow us to answer these questions and test our ideas

Chemistry		**Hydraulics**
$3A \Leftrightarrow 2B$ *A, B* different substances		
amount of substance		volume of water
difference of chemical potential		difference of pressure
$\Delta\mu \rightarrow$ Reaction rate		$\Delta p \rightarrow$ Water current
$\Delta\mu = 0$	*equilibrium*	$\Delta p = 0$

Figure 10: The analogies between hydraulics and chemistry.

Figure 12 shows the model results: we see that a certain quantity of A (upper curve) is transformed into B (lower curve): this occurs because under these initial conditions, the difference of the chemical potential for the reaction "A changes into B" assumes a negative value. We also see that after a certain time, the "driving force" shrinks to zero, so that the reaction rate vanishes as the system has reached equilibrium.

Figure 11 – The model: the amount of substances nA and nB vary when the reaction rate is different from zero; this happens when there is a difference of chemical potential. The latter can be computed on the basis of the values of the chemical potentials of the different substances under the initial conditions and must be actualized, during the reaction, according to the instantaneous values of the respective concentrations.



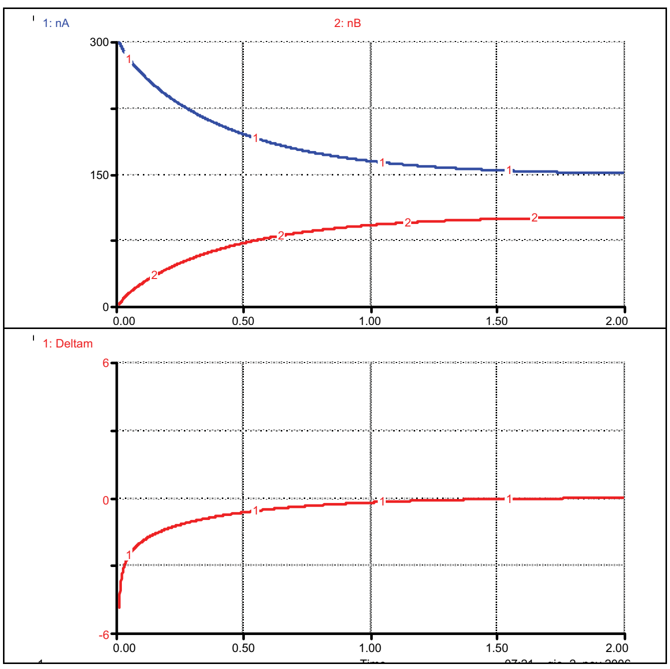


Figure 12 - Chemical reaction: the initial "driving force" decreases in magnitude and the system reaches equilibrium.

What kind of a process might the plots of figure 13 refer to?
At the beginning we have a difference in chemical potential with a positive value: the reaction that will occur is "B changes into A", as we can easily see in the lower plot. After about 3 seconds the system has reached a situation of equilibrium. But now something new happens: there is a sudden increase in the amount of substance A: by an external intervention to the system at this moment a certain quantity of A is added, so that the equilibrium is perturbed: there is now a surplus of A in the system, so that we again have a difference – with a negative value - in the chemical potential: this means that the reaction "A changes into B" must take place, until a new equilibrium is reached, approximately at the 6 second mark.

At this moment the equilibrium is again disturbed: a certain quantity of B is added to the system by an external intervention and again the system reacts in such a way as to reach equilibrium.

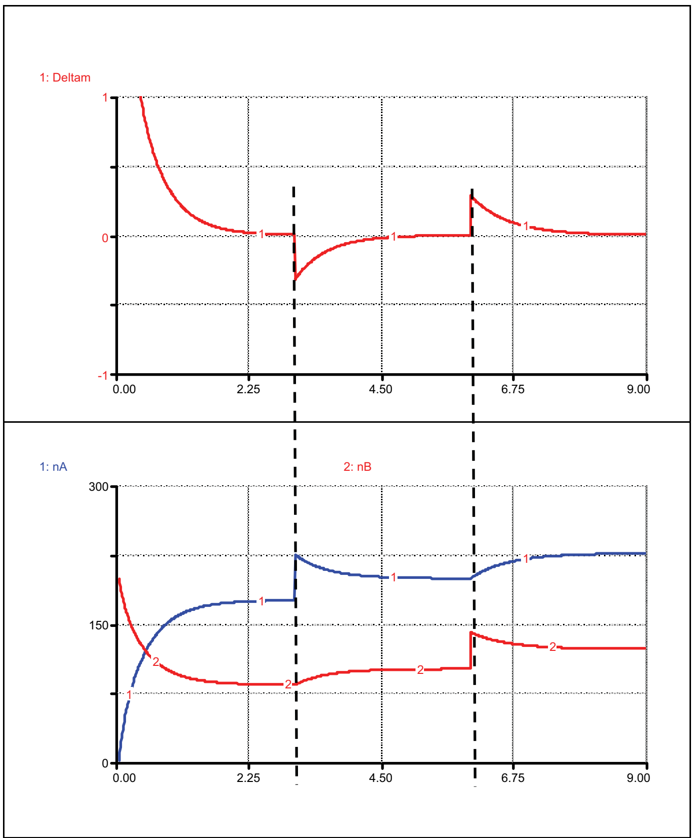


Figure 13 - Equilibrium and perturbation of the equilibrium.

Rather than as an illustration of the particular situation of the *Le Châtelier principle*, the interest of such examples lies in the possibility to introduce, to a certain extent, the factor time i.e. to recover dynamical aspects in chemical and biological processes as well.

3. Evaporation: A real experiment

In this third example I shall demonstrate the use of system dynamics modeling in the interplay between experiment and theory. Real life situations commonly confront us with a need to alternate between experimental and modeling approaches. Moreover, real applications often lead to models for which analytical solutions do not exist or are hard to come by. With system dynamics modeling not all is lost.

Rather than on the details of this evaporation experiment we will focus our attention on a possible strategy to put students in the condition to create step by step a satisfactory model for the real situation.
The experimental situation is the following (Figure 14): a certain quantity of water at room temperature is poured into a porous vessel. The vessel is placed on a balance, in order to measure the mass lost due to evaporation. With a couple of thermometers it is possible to record the temperature inside the vessel as well as the room temperature. A mixer keeps the water temperature as homogeneous as possible. The experimental results are shown in Figure 15.

Figure 14 - The apparatus for the evaporation experiment: (a) a vessel of porous material containing water; (b) three thermometers and a balance are connected to an on-line data logger; (c) a mechanical mixer keeps the water temperature as homogeneous as possible.

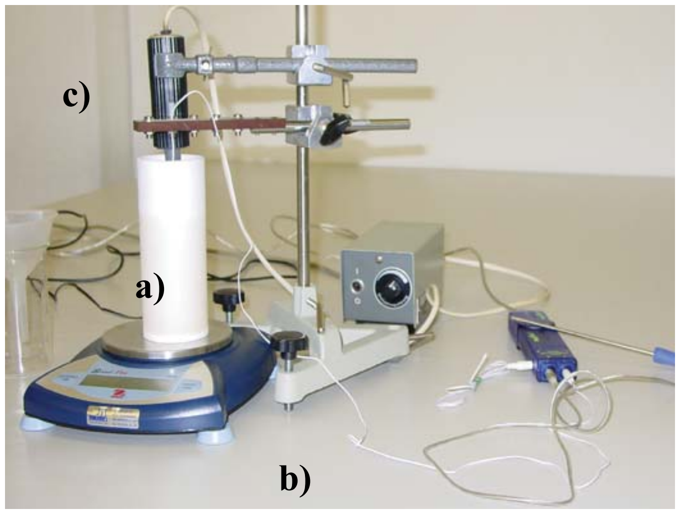
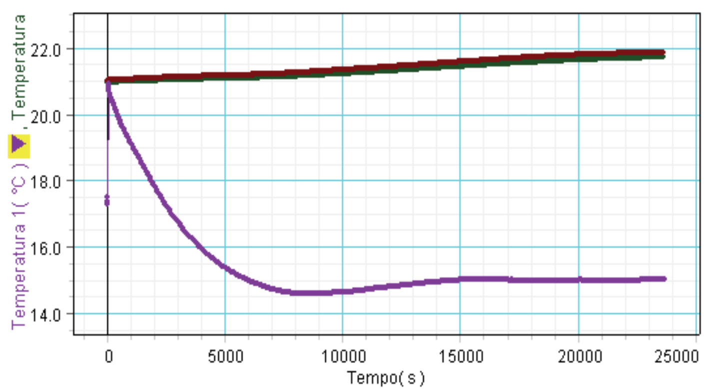


Figure 15 - Experimental result. The temperature readings plotted against time: above the room temperature and below, the temperature of the water in the vessel, which decreases by about 6 degrees Celsius in about two hours, and the system then reaches a more or less steady state.



The modeling strategy consists in the construction of a series of models: step by step, the processes taking place in the real situation are introduced into the model. We start with a vessel containing just water; in the second step we allow water to leave the vessel through a pipe; in the third step, the decrease in water is due to evaporation, while the vessel is still considered thermally insulated; in step 4 we add thermal contact with the surroundings: in other words, we add the conduction of entropy through the wall of the container; in step 5 we consider the real situation: data is added to the model, the simulation results are compared to the experimental results, and missing parameters are determined.
The models are structured on three pillars:

- First, the *law of balance of the mass of water* in the vessel: the mass of the water changes as a result of evaporation. The mass flow is not modeled: we take the measured data to specify this quantity.
- Second, the *law of balance of entropy of water*: the entropy of the water in the vessel is not constant because of phase change, mass transport of water, and heating from the vessel.

- Third, the *law of balance of entropy of the vessel:* the entropy of the vessel changes because of heating from the environment, and heating of the water.

Figure 16 shows the model and the related equations: some of the experimental parameters are obtained by matching the simulations results with the measured data. Figure 17 reports the final result.

4. Conclusions and perspectives

I hope you have been able to gain some insight into what students can do, learn, produce and, hopefully, understand. At the beginning of my paper I posed the question: is it possible to involve high school students in modeling activities? I shall begin with a half-answer to this question: Yes, it is, but only under certain conditions. Let me explain.

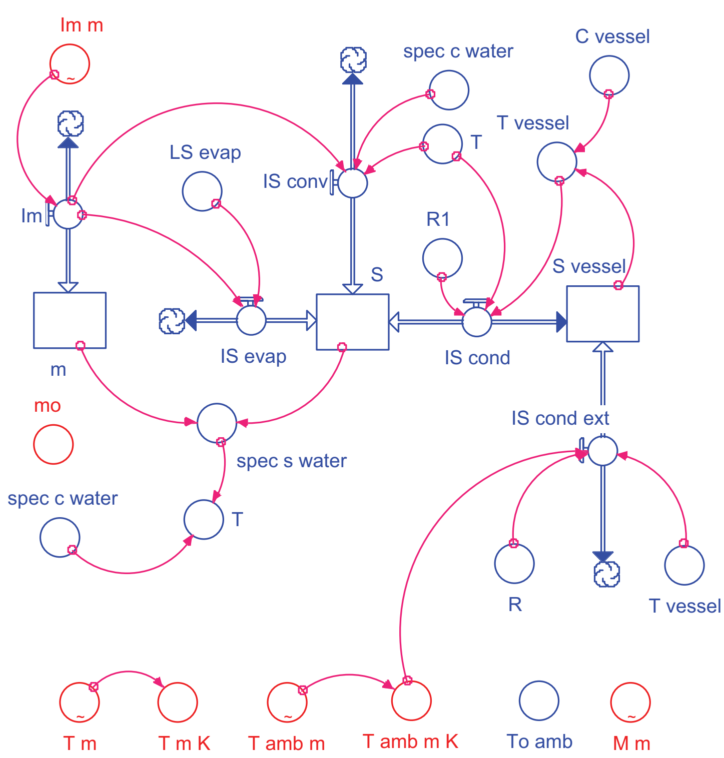


Figure 16 - The complete model for the real situation: the core consists in the entropy balance for the water contained in the vessel. Three possible causes for changes are considered: the endothermic character of evaporation; the convective transport associated with the outgoing water-flow; the thermal contact with the walls of the vessel. In this model, the entropy production due to heat transfer through the wall has not been taken into account. Measured data sets (indicated by circles marked with a ~) have been introduced into the model for water temperature and room temperature as well for the mass flow.

m(t) = m(t - dt) + (Im) * dt INIT m = mo INFLOWS: Im = -Im m S(t) = S(t - dt) + (IS conv + IS evap + IS cond) * dt INIT S = spec c water*mo* LOGN(To amb/ T ref) INFLOWS: IS conv = Im*spec c water*LOGN(T/ T ref) IS evap = Im*LS evap IS cond = -(T-T vessel)/R1 S vessel(t) = S vessel(t - dt) + (IS cond ext – IS cond) * dt INIT S vessel = C vessel*LOGN(Toamb/ T ref) INFLOWS: IS cond ext = -(T vessel-T amb m K)/R OUTFLOWS: IS cond = -(T-T vessel)/R1	C vessel = 250 {J/K} LS evap = 0.793e+4{ (J/K)/ kg)} mo = 0.184 {kg} R = IF TIME< 15000 THEN 928 ELSE 870 {K2/W} R1 = 17.4 {K2/W} Spec c water = 4184 {J/(K kg)} spec s water = S/m T = Tref *EXP(spec s water/spec c water) To amb = 294.2 {K} T amb m K = Tamb m+ T ref T m K = T m+ T ref T ref = 273.15{K} T vessel = T ref *EXP(S vessel/C vessel)

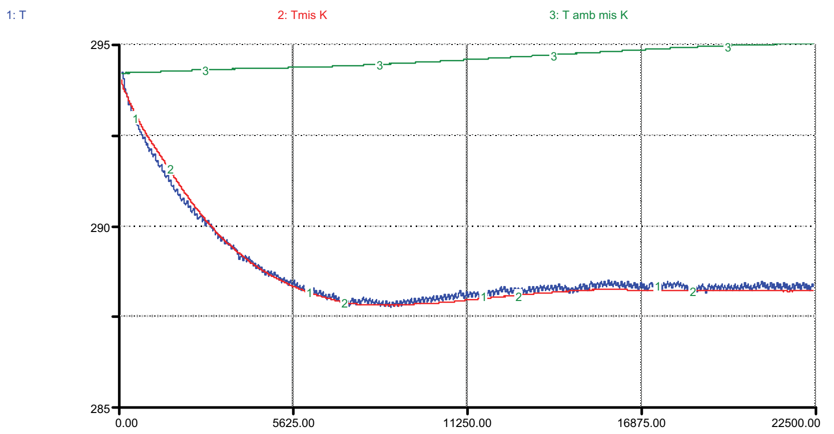


Figure 17 - The comparison between the measured temperature (curve 2) and the model prediction (curve 1): the result is satisfactory. Curve 3 represents the room temperature.

Students like the modeling activities: they are motivated and much more active than usual; they co-operate with each other. From the didactical point of view: the time necessary for the introduction of the basic operative procedures is reasonably short; the modeling activity permits individualized learning. Further, by focusing on the structure which underlies the functional relationships involved in a given situation, modeling also fosters the acquisition of transfer skills and interdisciplinary skills.
The cognitive acts carried out by the students are particularly interesting because the students are required to consider not just singular, isolated notions, but rather a whole network of connections, a veritable conceptual map informed by quantitative aspects. *This is true above all for work-environments which offer a graphic interface.*
However, some conditions must be met. Like all other activities involving new technologies, modeling should not become an end in itself, but rather a tool in the framework of a well defined didactical project. This requires a revision of the contents at the disciplinary level and also, perhaps, of some of the teaching objectives. Therefore we have to:

- prepare and develop suitable didactical materials;
- provide adequate teacher training;
- set up interdisciplinary discussion groups to promote and ensure substantial co-ordination with the other scientific disciplines and with mathematics.

Let me just close, then, by quoting Arnold Arons [3]: *"Wider understanding of science will be achieved only by giving students a chance to synthesize experience and thought into knowledge and understanding."* I am convinced that modeling can be a useful tool to this end..

References

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