Conceptual understanding of the Maxwell wheel motion

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Abstract

The concrete experience with the yo-yo wheel motion can become a good starting point for the student understanding of the concepts of rotational motion.
The first step is to gain information about student's knowledge in the field of rotational motion of flywheels with the help of conceptual question test. The results of the test, administrated to University students after the Mechanics course presented, show rather low level of understanding of the concepts of rotational motion.
In order to help in better understanding of the rotational motion concepts there is an inquiry based laboratory work designed for students. The aim of this labwork is to study the rotational and translational motion of the Maxwell wheel in order to describe and analyze the kinematics and dynamics of this kind of motion. Students measure and analyze the position, velocity and acceleration of the moving wheel with the help of MBL tools. Furthermore, they determine the force acting on the wheel at the lowest point and study the motion from the energetic point of view. They discuss the mentioned problems with their peers in order to draw their predictions into the working sheets. After the measurement they compared the gained results with their predictions.
The next part of the measurement is devoted to the study of the motion of two Maxwell wheels with the same mass but different moment of inertia. Their predictions can be compared with the results of real measurement or videomeasurement and the conclusions are presented. There are some additional problems to solve, i.e. Is there any final speed in case of a very long string or the speed will be increasing unlimitedly? The increase in moment of inertia leads to the decrease in acceleration of the wheel. Is there any limit in acceleration decrease? Is the force acting at the lowest point the same for the wheels of the equal mass and different moment of inertia? What is the length of the string for which the string tore apart assuming we know the breaking point of the material?
In order to evaluate the effect of the labwork on student knowledge the students answer a post-test about two weeks after the teaching.

Everyday experience and students first concepts

Everyday life offers a lot of occasions to observe the behavior of objects that present different physical phenomena. Popular child toy yo-yo wheel is an example of rotational motion of a flywheel caused by the presence of the gravitational force. This concrete experience with the yo-yo wheel motion can become a good starting point for the student understanding of the concepts of rotational motion. From the yo-yo wheel we can go on to the description of flywheels motion, e.g. to the simplified yo-yo used in the introductory physics lab called the Maxwell wheel.
Showing and presenting the physical factors influencing the objects functioning in the process of teaching can help in:

- Linking the already known and new students´ knowledge.
- Developing of physical thinking from concrete to abstract.
- Increasing students´ interest in physics.
- Presenting and understanding of the use of physical knowledge in everyday life.

Conceptual test questions about the rotational motion

In order to get information about the level and extent of students´ knowledge about the rotational motion of a flywheel we prepared a conceptual test. Students answer a set of 11 multiplied choice questions in about 10-15 minutes. The test is answered after the mechanics course before the laboratory work realized in MBL. The test items are aimed at:

- Description of the wheel velocity during the downward motion
- Description of the wheel angular velocity during the downward motion
- The lowest point of the motion
- The reason of the impulsive force at the lowest point of the motion
- The reason of the wheel's upward motion
- The influence of the wheel's mass on its speed
- The influence of the wheel's size (without any change in shape) on its motion
- The influence of the mass distribution (without any change in mass) on the wheel's motion

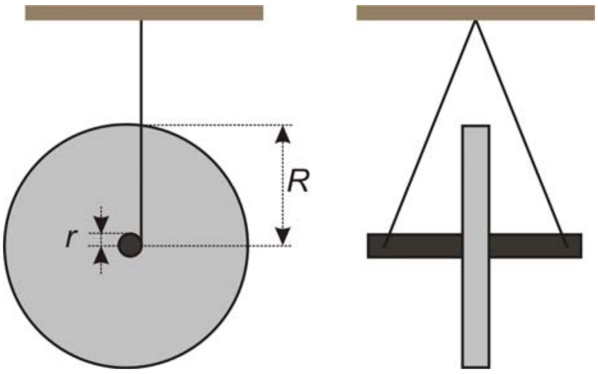
The test results show a low level in understanding of the principles of the flywheel motion. The most important points of misunderstandings are as follows:

- Understanding of two kinds of motion (translational and rotational).
- The influence of the moment of inertia on the wheel's rotational and translational motion.
- Understanding of the origin of the impulsive force acting during the change in direction of the wheel's motion.
- The influence of the mass distribution on the moment of inertia.

Theoretical model of the Maxwell wheel motion

The Maxwell wheel consists of a disc of radius R having an axis of radius r (with $r << R$) suspended from a fixed frame by two strings of equal length, which are wound around the axis (fig.1). There are two possible models describing its motion available. One is based on the energy consideration and the other one takes into account the fundamental laws of translational and rotational motion.

Fig. 1 Maxwell wheel



If we let the wheel fall from a height h, and the potential energy $U(x)$ is referred to the equilibrium position ($U = 0$ at $x = 0$), the system total energy E at the start ($x = h$, $v = 0$) is $E = mgh$. The force F responsible for the acceleration of the wheel is the gravitational force, which can be considered to act on the centre of mass of the body. During the fall the potential energy transforms into translational kinetic energy $E_t(x)$ and rotational kinetic energy $E_r(x)$. The energy of rotational motion is proportional to the moment of inertia of the wheel. Assuming negligible dissipation, we may write therefore:

$$E = U(x) + E_t(x) + E_r(x)$$

$$mgh = mgx + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

where m is the Maxwell wheel mass, I the moment of inertia and ω the angular velocity. Translational and rotational motions are not independent; the angular velocity is related to the linear velocity by:

$$v(t) = r\omega(t)$$

Solving with respect to the velocity, we get a relation similar to that for the velocity of a free body falling from height h:

$$v(x) = \sqrt{\frac{2g(h-x)}{1+\frac{I}{mr^2}}} = \sqrt{2\frac{g}{k}(h-x)} = \sqrt{2a(h-x)}$$

showing that the acceleration of the center of mass is:

$$a = \frac{g}{k} = \frac{g}{1+\frac{I}{mr^2}}$$

The same result can be gained using the dynamical equations describing both motions of the wheel. The net force acting on the wheel causes it to be accelerated according to the second law of motion

$$mg - F_t = m.a$$

where F_t is a sum of vertical string tensions acting upwards on the wheel.

The dynamical equation describing the rotational motion of the wheel is as follows:

$$T = F_t.r = I.\varepsilon = I.\frac{a}{r}$$

where T is a net applied torque and ε is an angular acceleration of the rotational motion. Combining these two equations we finally get the same expression for the acceleration of the wheel.

From the result for the wheel acceleration it can be seen that the increased moment of inertia results in decrease in acceleration of translational motion. For $r \ll R$, the moment of inertia, I, of the wheel is a good approximation to that of the disc ($I \approx mR^2/2$), and therefore the predicted acceleration is

$$a \approx \frac{g}{1+\frac{R^2}{2r^2}} \approx g\frac{2r^2}{R^2}$$

The elastic energy of the stretched string has not been taken into account. During upward and downward motion the total energy decreases. At the lowest point of the motion there is a change of linear velocity sign (passing through zero) while the angular velocity remains approximately constant. There must be a force the string acts on the wheel. As a reaction there is a force that the wheel acts on the string that we can feel during the velocity change at the lowest point.

Teaching method

In order to help in better understanding of the rotational motion concepts there is an inquiry based laboratory work designed for students. The aim of this labwork is to study the rotational and translational motion of the Maxwell wheel in order to describe and analyze the kinematics and dynamics of this kind of motion. Students working with their peers can realize two types of measurements depending on the apparatus available. They can do a real-time measurement on the Maxwell wheel with the help of an ultrasonic motion detector and a force sensor or a video measurement on a video clip already prepared for the students (fig.2). After collecting the data the analyzing procedure is the same in both cases. Measuring the position of the wheel and the string tension in the case of real-time measurement students analyze the results according to the instructions in their working sheets creating velocity vs. time and acceleration vs. time diagrams from the position vs. time diagram (fig 3).

Fig.2 Video measurement realized in IP COACH system.

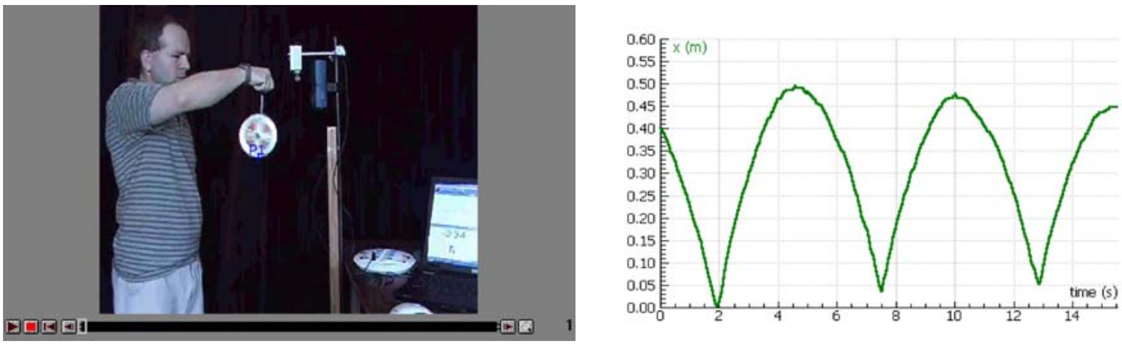
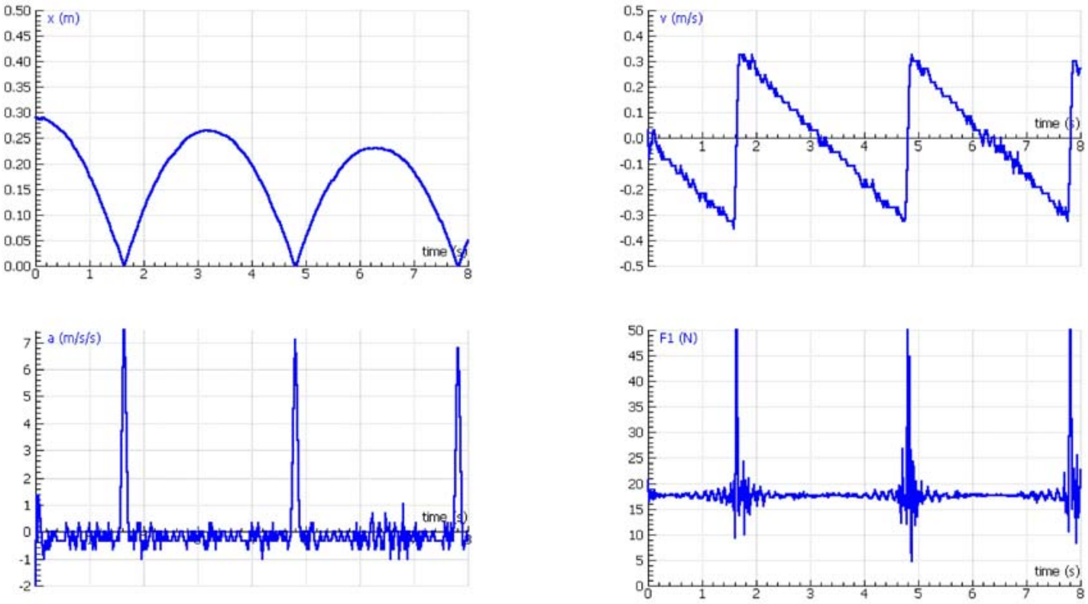
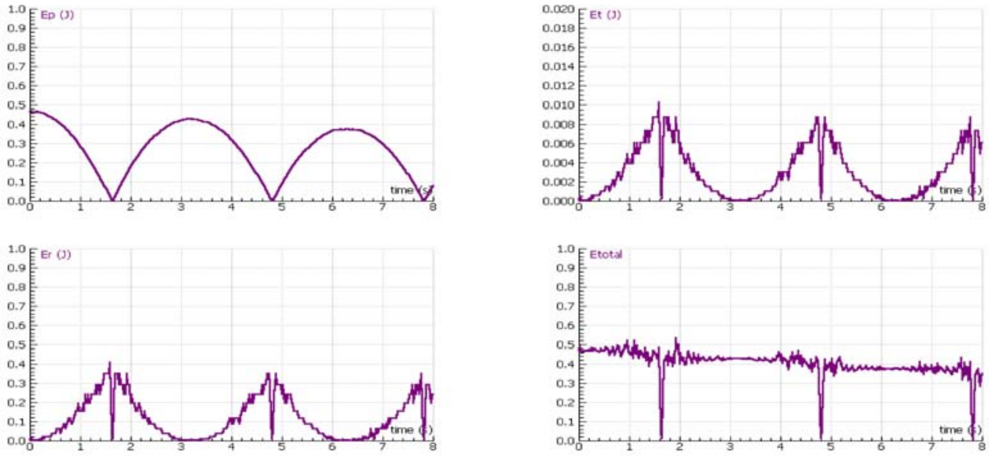


Fig.3 Position vs. time graph, velocity vs. time graph, acceleration vs. time graph, string tension vs. time graph gained from the real-time measurement.



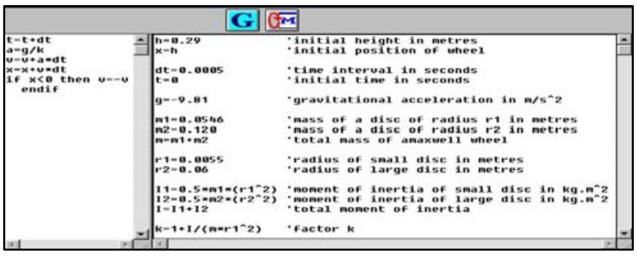
Furthermore, they can determine the force acting on the wheel at the lowest point with the help of the acceleration counted from the change in velocity. The analysis can be followed by the energy balance of the motion (fig.4). They discuss the mentioned problems with their peers in order to draw their predictions into the working sheets. After the measurement they compare the gained results with their predictions and with the theoretical results (fig.5).

Fig.4 Potential energy vs. time graph, translational kinetic energy vs. time graph, rotational kinetic energy vs. time graph, total energy vs. time graph gained from the real-time measurement.



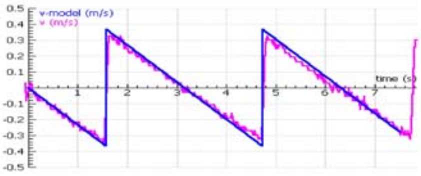
The next part of the measurement is devoted to the study of the motion of two Maxwell wheels with the same mass but different moment of inertia. Their predictions can be compared with the results of real measurement or video measurement and the conclusions are presented. In order to evaluate the effect of the labwork on students´ knowledge the students answer a post-test about two weeks after the teaching.

Fig. 5 Theoretical model of the Maxwell wheel motion (left) and the velocity vs. time graphs (theoretical and experimental).



```
t=t+dt
a=g/k
v=v+a*dt
x=x+v*dt
if x<0 then v=-v
  endif
```

```
h=0.29                'initial height in metres
x=h                   'initial position of wheel

dt=0.0005             'time interval in seconds
t=0                   'initial time in seconds

g=-9.81               'gravitational acceleration in m/s^2

m1=0.0546             'mass of a disc of radius r1 in metres
m2=0.120              'mass of a disc of radius r2 in metres
m=m1+m2               'total mass of amaxwell wheel

r1=0.0055             'radius of small disc in metres
r2=0.06               'radius of large disc in metres

I1=0.5*m1*(r1^2)      'moment of inertia of small disc in kg.m^2
I2=0.5*m2*(r2^2)      'moment of inertia of large disc in kg.m^2
I=I1+I2               'total moment of inertia

k=1+I/(m*r1^2)        'factor k
```



Student’s worksheets

All the instructions for the students are presented in the working sheets. There are questions to answer, discussions, prediction sheets and conclusion sheets prepared for the students to fill in. Students gradually discover the properties of the flywheel motion and the results gained experimentally can be compared with the theoretical results (fig.5). After the labwork we can verify the level of their knowledge with the help of complementary questions, such as:

- Is there any final speed in case of a very long string or the speed will be increasing unlimitedly?
- The increase in moment of inertia leads to the decrease in acceleration of the wheel. Is there any limit in acceleration decrease?
- Is the force acting at the lowest point the same for the wheels of the equal mass and different moment of inertia?
- What is the length of the string for which the string tore apart assuming we know the breaking point of the material?

Conclusion

The designed labwork aimed at the study of the behavior of a simplified model of the well-known child toy yo-yo (Maxwell wheel) realized with the help of MBL tools (PC with interface and sensors or videoanalysis) offers students to realize inquiry-based learning in MBL. Furthermore, it allows an opportunity to compare theoretical results with the ones gained from the experiment. The labwork itself is supported by the worksheets with the instructions that should help students to concentrate to the most important points of the wheels´ motion to study and analyze. We are persuaded that this way of teaching and learning can surely help in better understanding of the presented phenomena and in changing the misunderstandings and misconceptions the students have. In the future we are planning to continue in this work in order to help students in better understanding of the physical phenomena that are proved to be difficult to understand in standard physics courses.

Acknowledgments

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