

Various Physics

Modelling dynamical equilibrium in emission and absorption of radiation

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Abstract

A very simple experiment often programmed already in primary schools is to observe the temperature rise of a black object exposed to solar radiation and to compare it with the temperature rise of an identical white object exposed in similar conditions.

The physics of this simple experiment is however much richer than one generally realizes at this basic level. For example, it is not common to compare also the rate of cooling of the two objects: one observes, if the measure is done correctly, that the black object cools faster than the white one, as expected from Stefan Boltzmann and Kirchhoff laws [1]. It is also very instructive to measure how the rate of temperature rise slowly decreases in time until a *dynamical equilibrium temperature* is reached: when this happens, the rate of energy absorption is equal to the rate of energy emission and both the equilibrium temperature and the relaxation time depend critically on the type of object and of the experimental conditions.

In the paper we discuss results obtained by high school students during a stage in our university lab and a simple model developed to interpret the data in terms of dynamical equilibrium between absorption and emission of radiation, which shows the complete similarity between the two processes.

The experiment

The experimental setup is shown in figure 1. It consists of two identical metal cans, one painted white and the other black, and of an incandescent high wattage lamp to provide the radiation symmetrically to the two cans. To record the temperatures the students could use either manual thermometers or online ones, with the readout through an ADC converter and a graphic calculator.

Typical results are shown in figure 2. The lamp was on until the temperature rise of both cans seemed negligible, indicating that the dynamical equilibrium between the energy absorbed from the lamp and the energy released by conduction and radiation



Figure 1

emission was very near. The lamp was then turned off and the temperatures of the two cans were measured during the cooling process until the room temperature equilibrium was very near.

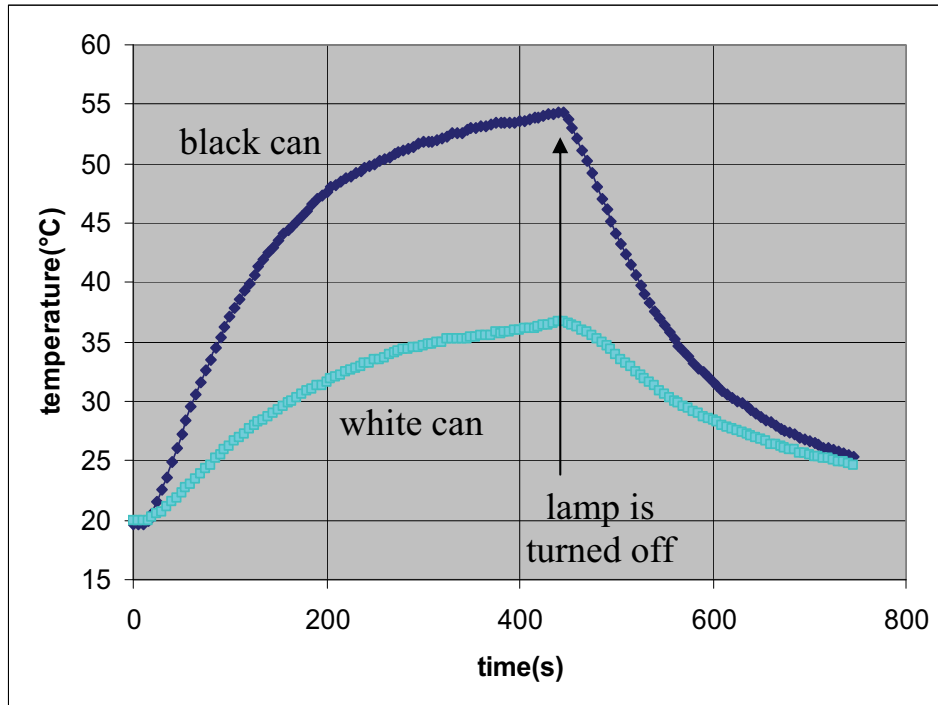


Figure 2

The curves obtained for both cans show a similarity between the heating and the cooling processes: in the heating process the temperature seems to approach a “dynamical equilibrium temperature”, which is clearly larger for the black can, and during the cooling process the temperature seems to approach a “static equilibrium temperature”, which should be the same for the two cans and close to the room temperature.

The model

Modelling the evolution of a physical process is a powerful method to understand it, as is well known from educational research [2]; cooling and heating phenomena are particularly suitable [3], because the model helps to organize the spontaneous reasoning which is natural for such familiar processes.

We modeled the approach to equilibrium with an *exponential law*, both for the static and for the dynamical case. During heating, the difference between the “dynamical equilibrium temperature”, T_{dyn-eq} , and the temperature T at time t is assumed to decrease exponentially in time with a characteristic relaxation time τ_d :

$$\Delta T = T_{dyn-eq} - T = T_{od} e^{-t/\tau_d} \quad (1)$$

The data suggest T_{dyn-eq} to be clearly larger for the black can than for the white can and to depend upon the power of the radiation source; it is

more difficult to visually compare the dynamical relaxation times of the two cans, although it seems smaller for the black can.

During cooling, the difference between the temperature T at time t and the “static equilibrium temperature”, $T_{stat-eq}$, is assumed to decrease exponentially with a characteristic relaxation time τ_s :

$$\Delta T = T - T_{stat-eq} = T_{os} e^{-t/\tau_s} \quad (2)$$

The data suggest $T_{stat-eq}$ to be roughly the same for the two cans and close to the room temperature.

To determine the parameters of the model a simple “excel” work sheet was used: the temperature differences defined according to equations (1) and (2) for the two processes were plotted on a logarithmic scale as a function of time, using tentative values of the equilibrium temperatures, which were manually varied until a good fit to a decreasing exponential behavior was obtained.

The logarithmic plots are shown in figure 3 for the two cans and the best fit values are collected in table 1.

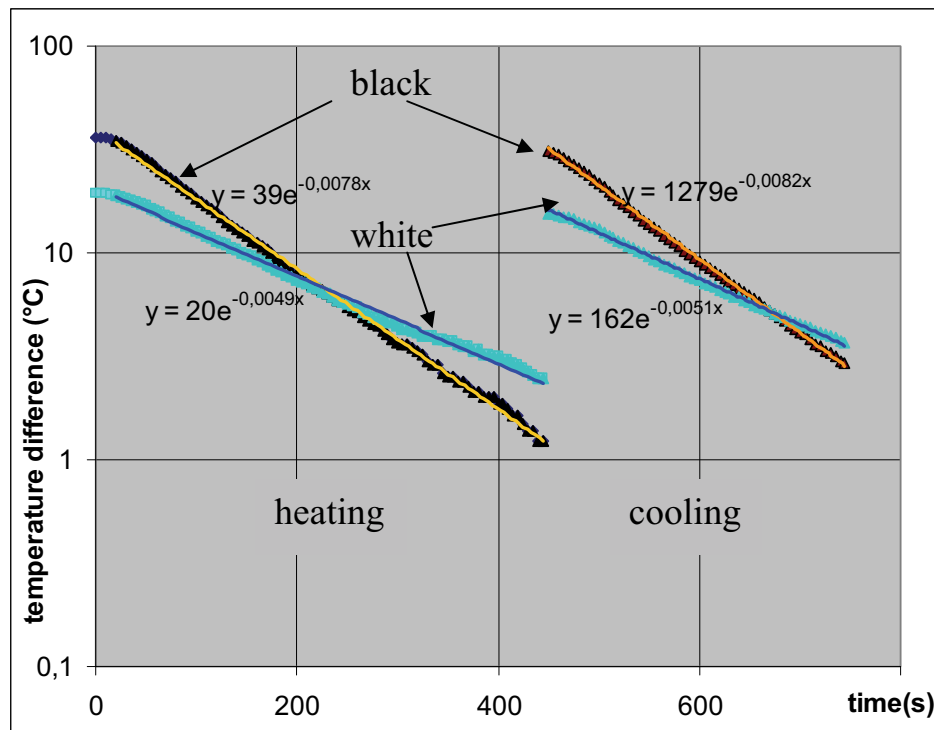


Figure 3

	$\tau_d \approx \tau_s$	T_{dyn-eq}	$T_{stat-eq}$
Black can	≈ 120 s	55,5 °C	22,5 °C
White can	≈ 200 s	39,1 °C	21,1 °C

Table 1

The main conclusions are:

- the model describes well both the heating and the cooling processes, showing that the underlying dynamics are the same: the fact that the equilibrium temperature is approached with a decreasing exponential law shows that the energy loss is roughly proportional to the difference between the present temperature and the equilibrium temperature¹⁸, both for cooling and for heating;
- the characteristic relaxation times τ_d and τ_s during heating and cooling are very similar, for both cans, showing that the underlying energy loss processes are the same, as expected from Kirchhoff law, that is the absorption and the emission of radiation are mirror processes; in particular the characteristic relaxation times τ_d during heating are independent of the power of the radiation source, contrary to what one would naively expect, since they are close to τ_s ;
- the characteristic relaxation times τ_d and τ_s of the black can are significantly smaller than those of the white can, as expected, since the black can is *more strongly coupled* with the environment;
- the dynamical equilibrium temperature of the black can is larger than that of the white can and both depend upon the power of the radiation source, as expected;
- also the static equilibrium temperature of the black can is larger than that of the white can, contrary to the naïve expectation: this fact suggested further investigation as we will discuss below.

The rate of the temperature variation

The above analysis indicates that the key variable of this kind of process is the rate of the temperature variation, because, according to the model, it should show more clearly the mirror exponential behavior of the heating and cooling processes:

$$\frac{dT}{dt} = -\frac{d(T_{dyn-eq} - T)}{dt} = -\frac{d(T_{od}e^{-t/\tau_d})}{dt} = \frac{T_{od}e^{-t/\tau_d}}{\tau_d} \quad \text{during heating (3)}$$

$$\frac{dT}{dt} = \frac{d(T - T_{stat-eq})}{dt} = \frac{d(T_{os}e^{-t/\tau_s})}{dt} = -\frac{T_{os}e^{-t/\tau_s}}{\tau_s} \quad \text{during cooling (4)}$$

¹⁸ For the radiation one would indeed expect a proportionality to T^4 , but, for the small temperature differences involved in the experiment, the linear approximation works well.

In figure 4 we show the plots of the temperature variation rate of the two cans as a function of time, where the “mirror” behavior is directly appreciable.

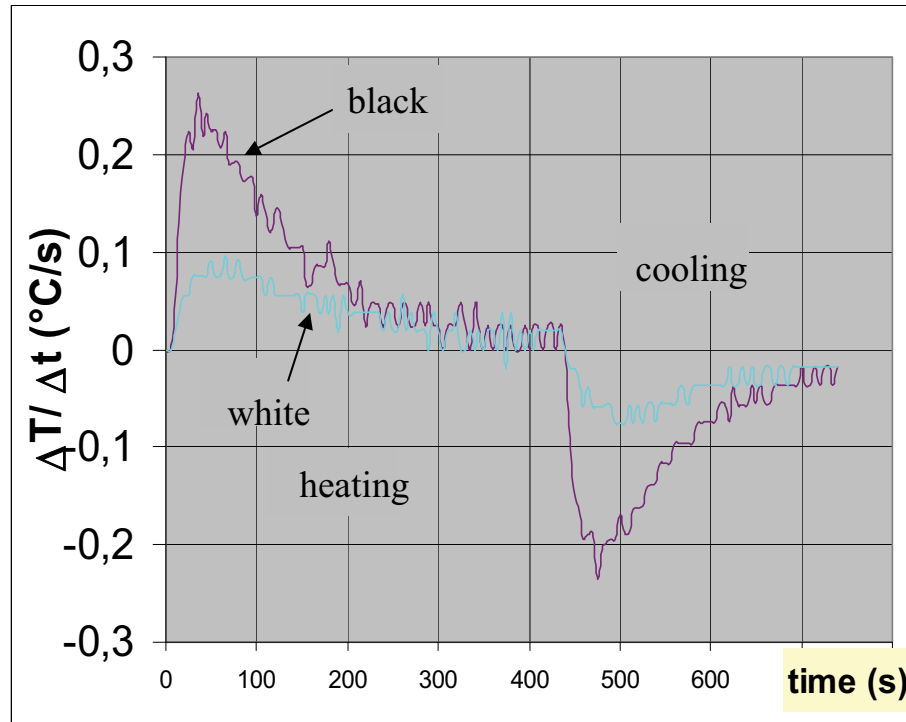


Figure 4

Equations (3) and (4) show also that the temperature variation rate is a linear function of the temperature with slope $1/\tau$.

$$\frac{dT}{dt} = \frac{T_{od}e^{-t/\tau_d}}{\tau_d} = \frac{T_{dyn-eq} - T}{\tau_d} \quad \text{during heating} \quad (5)$$

$$\frac{dT}{dt} = -\frac{T_{os}e^{-t/\tau_s}}{\tau_s} = \frac{T - T_{stat-eq}}{\tau_s} \quad \text{during cooling} \quad (6)$$

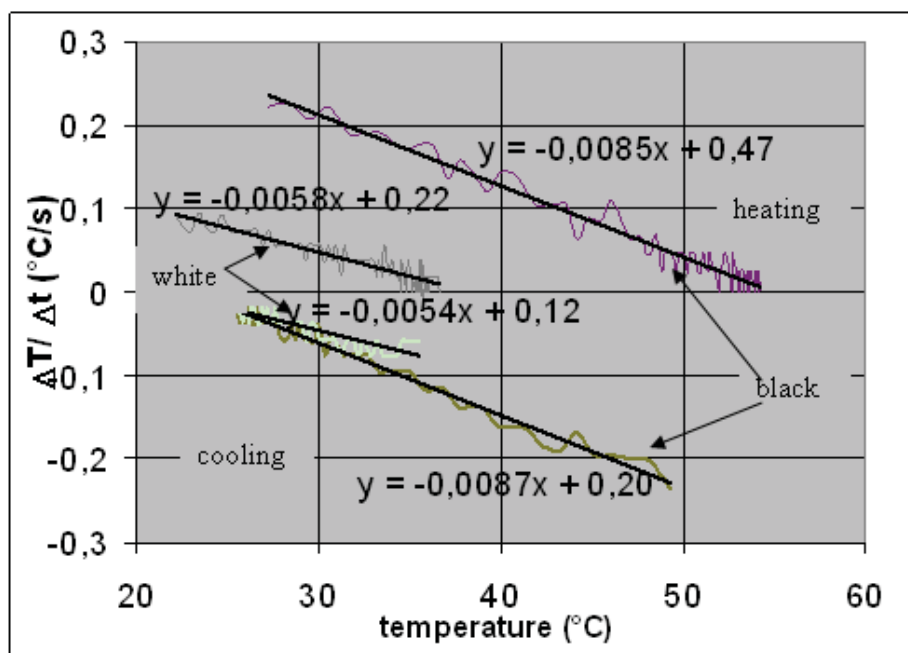


Figure 5

From the plot, shown in figure 5, of the temperature variation rate as a function of temperature, we can thus determine with a simple linear fit all the parameters of the model, which are collected in table 2.

Table 2

	τ_d	τ_s	T_{dyn-eq}	$T_{stat-eq}$
Black can	118 s	115 s	55,3 °C	23,0 °C
White can	172 s	185 s	37,9 °C	22,2 °C

The best fit values are in complete agreement with those obtained in the previous analysis and therefore confirm the conclusions presented above.

A home made radiometer

A puzzling result of the above experiment is that the static equilibrium temperature appears to be slightly larger for the black can than the white can. Since similar differences were obtained also when repeating the experiment in different conditions, we concluded that the effect was real and that the black can was more sensitive than the white one to the presence of small amounts of IR radiation present in the lab. We thus prepared a “home made radiometer” using two small black painted metal strips fixed at the two opposite sides of a cube made of insulating material (figure 6). Having a smaller mass, this “radiometer” is more sensitive than the black can to small variations of IR radiation and also it is more sensitive to the direction of the IR radiation, which is an important feature to disentangle energy absorption by radiation and by conduction. In the figure the radiometer was put on a table in the garden, with one side facing the house wall and the other facing the garden, just before sunrise: the two thermometers indicate a small but significant difference between the two temperatures, indicating that more IR radiation was coming from the wall. As the sunrise advanced, the temperatures slowly changed and the difference was completely reversed much before direct sun light hit the outer thermometer!

Conclusions

This simple experiment, which can be performed with basic instrumentation, is very instructive to show the unique features of the energy exchange by radiation and the importance of a good mathematical model to correctly interpret the dynamics of the

Figure 6



coupling with the environment which allows reaching the equilibrium conditions.

References

- [1] see for example P.A.Tipler, Physics for Scientists and Engineers, World Publishers (1991); M. Alonso and E.J. Finn, Physics, Addison Wesley (1992)
- [2] R. Sperandeo, Learning physics via model construction, GIREP Seminar “Developing Formal Thinking in Physics”, Udine 2001, pag.117 (2002)
- [3] R. Sperandeo and M.L. Aiello-Nicosia, A model of pre-service teacher training: hands-on experiments and hands-on activities, ICPE-GIREP Conference “Hands-on experiments in physics education”, Duisburg 1998, pag.339 (1999)

Acknowledgments

We wish to thank **Ciro Marino** for technical support and **Stefania Bressan** and **Albert Werbrouck** for helpful discussions.