Modeling in Non-linear Physics

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Abstract

In the study of complex dynamical systems, topics as chaos, fractal analysis, self organized criticality (SOC), non-stationary time series analysis, and others are emergent. Non-linear dynamics is a new way of applying the known laws of Physics, with the aid of the computer, to many phenomena that encompass, in addition to the traditional in Physics, phenomena of Biological and Social Sciences. In this work, we review some models that have been important to understand many concepts of non-linear dynamics, as the sand pile model to understand SOC concepts: By using this basic model we can study forest fire models, epidemic models or we can build spring-block models to mimic the dynamics of a seismic fault. We also review some models applicable to Physiology. For the study of these and many other models the student needs University Physics, besides he needs to know the system basics that he is studying (for example, the basics of the heart Physiology if he is working with models of the heart dynamics) and he also needs the computer. We can teach to the student basic models and later he can do modifications to model new situations, very often these applications are very illustrative and interesting. These topics have not been approached in the university Physics courses and its study has been postponed to the graduate courses, we propose to include them in the undergraduate Physics curriculum. The undergraduate student has all the elements to work with success many of these non-linear basic models.

Introduction

Some decades ago we only studied phenomena that obeyed integral equations, in particular the linear ones. If a non-linear phenomenon was studied it was transformed by means of a linear approach. It seemed that in Nature the really important thing was the family of linear phenomena and that the other ones were an exception, they were undesirable for the difficulty of their treatment. It seems that now it begins to be accepted that the immense majority of the natural phenomena are not linear, and that the other ones are the exception.

Non-linear dynamics

Many concepts of the non-linear dynamics have arisen, some of which surely are known by the students, for instance: Power laws in Physics, Physiology and other areas; random walks in the stock market and under the microscope; floods, forest fires, galaxies distribution, and other cases with statistical auto-similarity. Cantor and Julia's sets have been popularized in the Internet and the term "art fractal" has been coined. In fact, fractals and their characterization using fractional dimensions are now very popular.

Chaos

In the moment of its discovery, the phenomenon of chaotic movement was considered a mathematical rarity. In 1963, Edward Lorenz had a mathematical basic program to study a simplified model of the climate. Since the code of the computer was deterministic, Lorenz thought that introducing the same initial values, he would get the same result when executing the program several times. Lorenz was surprised because drastically different results were obtained every time. For the limitations of his equipment, lightly different values were introduced. This principle sometimes is called the "Butterfly Effect".

Non-linear dynamics and complexity

Non linear-dynamics is a new and promising way to apply the well-known laws of Physics, with the fundamental help of computers, to very varied phenomena that embrace, besides the traditional ones in Physics, those that show up in the Biological Sciences and the Social Sciences. Complexity designates the study of dynamic systems that are in some intermediate point among the order and total disorder. These systems become extremely sensitive to their initial conditions. One of the more interesting results due to the emergence of this new research field is that they have formed interdisciplinary research groups to study the inherent problems of dynamic complex systems. Complexity of a system should not be confused the fact that a system is complicated. In fact, we should speak of complex behavior of a system, because a dynamic system can be very simple but it can exhibit under certain conditions an unexpected behavior of very complex characteristics.

Some applications and models

There are applications of the non-linear dynamics to Physics, Chemistry, Biochemistry, etc. Many topics of non-linear dynamics are important research themes of the scientific community, some of them have been successful in explaining complex behaviors observed in nature, some of them have had only partial success. Among the last, it can be mentioned the self-organized criticality (SOC) that tried to explain some ubiquitous patterns that exist in the nature, among them fractal structures and catastrophic events. According to Per Bak [1], SOC can explain massive extinctions. The concept was proposed in 1987, the basic idea is simple and most of the mathematical models that have been used in the implementation of the theory are not complicated. Almost anyone that knows basic programming and with a PC can implement the models to verify their predictions. Another concept that has been mentioned much lately is the one that refers to multifractal analysis [2, 3, 4]. Although at the beginning multifractals were perceived as an isolated surrounded island regulated by an occult formalism, lately there have been important advances in the popularization of the related concepts and the multifractal analysis has been constituted as a very useful tool in the analysis of certain type of time series. They have been identified in different fields in independent form and they have been named in different ways. In Physics and Engineering this phenomenon is called 1/f noise. Others talk of non-Fickean Diffusion. The turbulence specialists associate some instances of the phenomenon with intermittence.

Time series analysis

The most direct league between chaos theory and the real world is the analysis of time series of real systems in terms of non-linear dynamics. Traditionally, stochastic linear processes have modeled the non-periodic signals. But inclusive the dynamic simplest systems can exhibit temporary strongly irregular evolution. Chaos theory offers new concepts and algorithms for the study of time series that can take to a better understanding of signals. There are new concepts and methods as the Lyapunov exponents, noise reduction, non-linear prediction, dimensions and entropies as well as statistical tests for the nonlinearity. Others are control chaos, wavelet analysis and pattern dynamics [5, 6].

The theory of non-linear dynamical systems provides new tools and quantities for the data characterization of irregular time series. The analysis of time series has physiologic important applications, for example, starting from the analysis of interbeat heart time series [3, 4] some heart anomalies can be recognized.

An example, modeling the dynamics of a seismic fault.

The theory of plate tectonics says that the lithosphere is broken into about a dozen major rigid plates and several minor ones. These plates slowly grind against each other, building up stress and cresting faults. Seismologists have observed that small quakes occur more frequently than large quakes. Gutenberg and Richter established a scaling relation between the magnitude and the frequency of earthquakes. The Gutenberg and Richter relation is $log_{10}N(m) = a - bm$, where a and b are constants and $N(m)$ is the number of earthquakes greater than m in a specified time interval [7, 8]. A first check on the robustness of an earthquake-fault model is that it be able to produce these scaling relations. However, the ability to produce a scaling relation does not mean that the model is useful, because it also must be able to reproduce other known phenomena and led to predictions that the seismologists can observe on real faults.

Seismic faults models

The Olami, Feder and Christensen (OFC) model was proposed in 1992 [7]. The OFC model is a non-conservative cellular automaton model for describing the dynamics of a 2-D array of rigid blocks on a frictional surface (Figure 1). It consists of an LxL array of individual blocks identified by (i, j), where i, j are integers between 1 and L. Each block is connected to its four nearest neighbors by springs with elastic constants

K_1 and K_2 and it is connected on its top to a moving driving plate by means of a spring with stiffness K_L. The displacement of each block from its relaxed position on the lattice is $X_{i,j}$ and the total force exerted by the springs on a block *(i, j)* is given by [7]

When the two rigid plates move relatively among them the total force in

$$F_{i,j} = K_1\left[2x_{i,j} - x_{i-1,j} - x_{i+1,j}\right] + K_2\left[2x_{i,j} - x_{i,j-1} - x_{i,j+1}\right] + K_L x_{i,j}$$

each block it is increased uniformly (with a rate proportional to $K_L V$, where V is the relative speed among the plates), until a site reaches a value limit and the relaxation process begins). The redistribution of forces after local slip at position *(i, j)* due to the force on one of the blocks is larger than the maximal static friction and is given by

$$F_{i\pm1,j} \rightarrow F_{i\pm1,j} + \delta F_{i\pm1,j}$$
$$F_{i,j\pm1} \rightarrow F_{i,j\pm1} + \delta F_{i,j\pm1}$$
$$F_{i,j} \rightarrow 0$$

where the increments in the force on the nearest-neighbor block are

$$\delta F_{i\pm1,j} = \frac{K_1}{2K_1 + 2K_2 + K_L} F_{i,j} = \gamma_1 F_{i,j},$$
$$\delta F_{i,j\pm1} = \frac{K_2}{2K_1 + 2K_2 + K_L} F_{i,j} = \gamma_2 F_{i,j},$$

γ_1 and γ_2 are called the elastic ratios, and for the case $K_L > 0$ the redistribution of the force is non-conservative, as is expected to occur in actual earthquakes. This redistribution redefines the forces in the nearest-neighbor blocks, and further slips can occur, causing a chain reaction (synthetic earthquake). For instance, in Figure 2, we show the surface rupture that constitutes a synthetic earthquake. We can count the relaxed sites so we can assign a magnitude to each earthquake, as it can be seen in Figure 3, we can repeat a lot of times the same procedure, therefore we have many events with different magnitudes, all of them form a time series of magnitudes (Figure 3). We can see that these synthetic earthquakes follow the Gutenberg-Richter law (Figure 4). This model and other modifications of the same model can reproduce qualitatively many features of real seismicity [8].

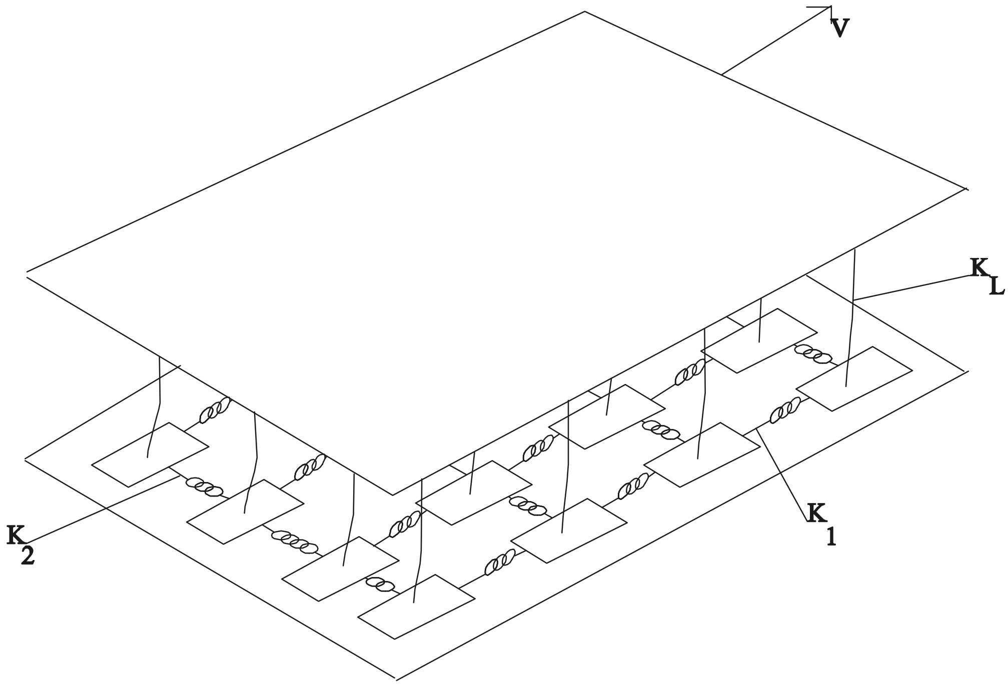

Figure 1. The geometry of the spring-block model. The force on the blocks increases uniformly as a response to the relative movement of the two plates.

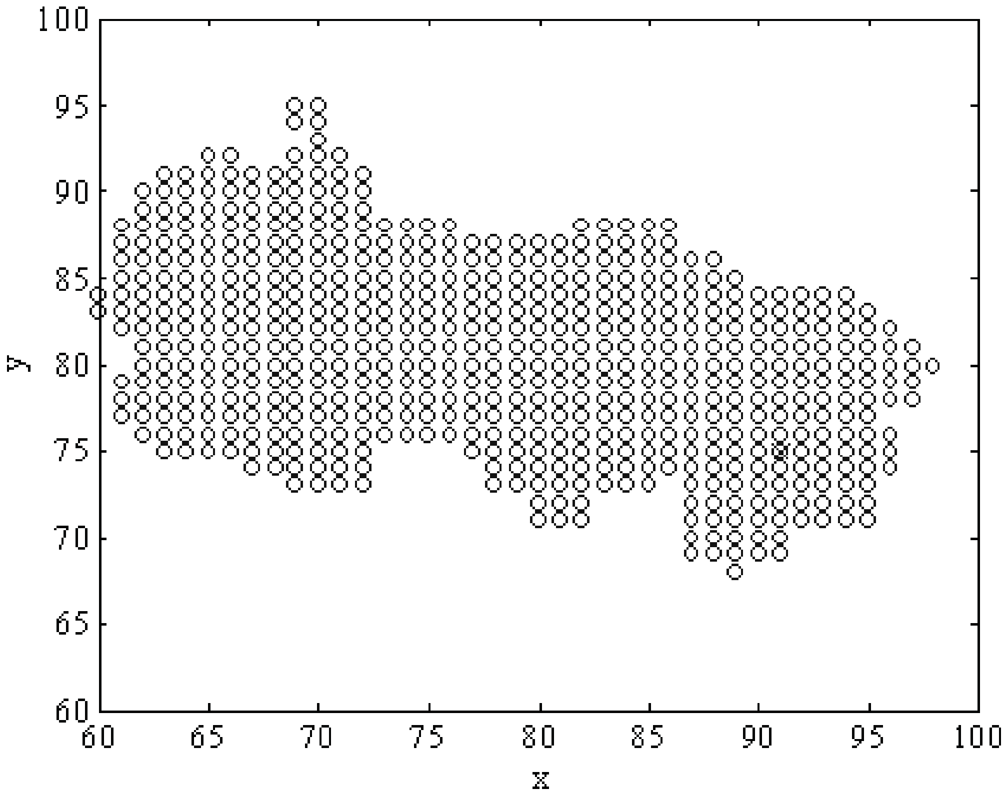

Figure 2. The rupture surface in a synthetic earthquake. The X point is the epicenter

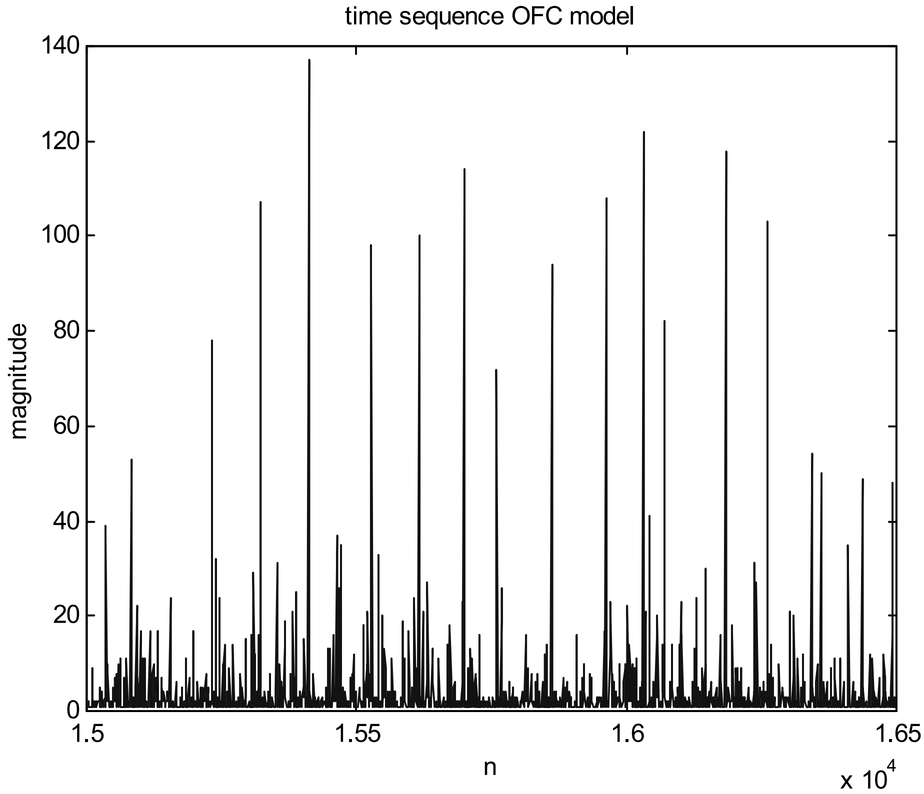

Figure 3. Time series of synthetic earthquakes (16384 events).

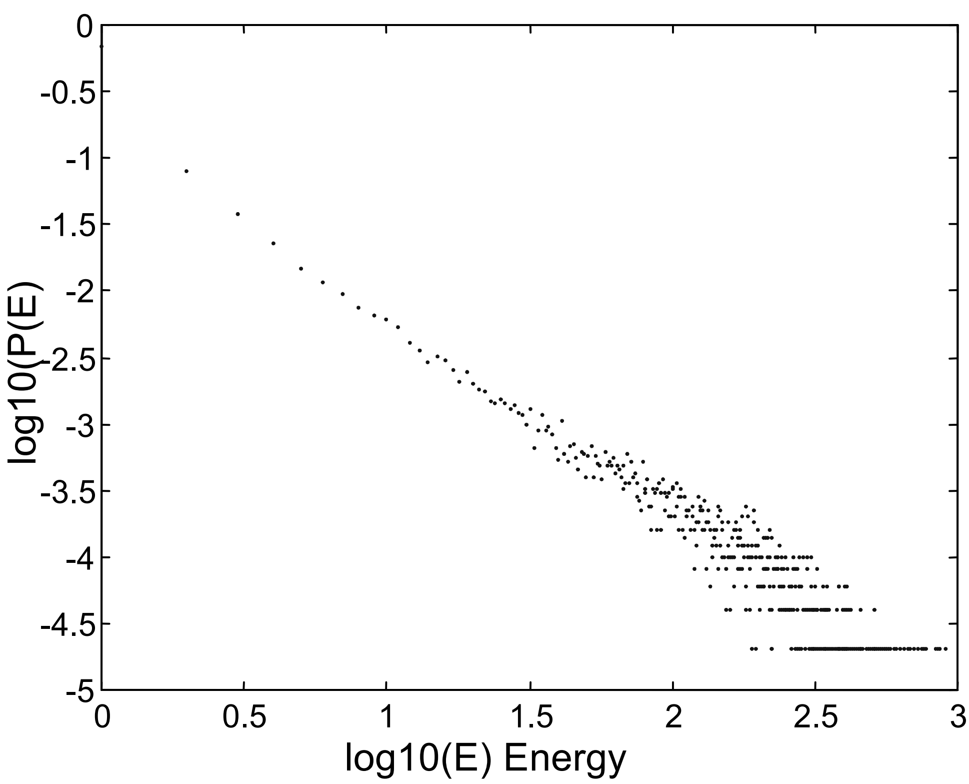

Figure 4. Gutenberg-Richter law for synthetic earthquakes.

Conclusions

The field is prepared to intend that subjects of non-linear dynamics are included in Physics undergraduate programs, there is already a lot of materials that can be used to make an effective approach to such topics at university level besides that there exist many divulgation materials. Including these topics would open the students a panorama to one of the Physics areas that has been advancing a lot in the last times. But the applications that are carried out are also intrinsically very interesting and now the theory has become accessible more than ever, so this proposal of including new courses can also extend to the Engineering careers, mainly those that have to do with physiologic systems, for example, we can intend a course of time series analysis for the biomedical engineers that have to analyze time series of multitude of physiological signals, most of which it has been shown that exhibit non-linear behavior.

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