At the Crossover between Physics, Mathematics, Astronomy and History: a Didactic Proposal on Römer's Measure of the Speed of Light

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Abstract

This contribution discusses the role of mathematics and history of science in physics education. Römer's experiment is interesting because it was part of the historical debate between the two conceptions of instantaneous propagation and finite speed of light. Moreover, studying it more closely than textbooks normally do forces the use of unobvious mathematic tools adaptable to different educational levels. The subject is presented with questions and suggestions regarding the relative movements of the four stars concerned (Sun, Earth, Jupiter, Io) and the occultations and eclipses of Io. Cabri software is used for pictures and animations, and astronomical tables for time calculations.

1 Mathematics and Physics

To physics, mathematics is not only a useful tool, but also an essential part of the structure and meaning of physical concepts and theories. According to Bachelard [1], the physical thought is, in essence, mathematic. Barrow [2] stressed that between mathematics and physics exists a symbiotic relationship. According to Piaget [3], "knowledge of the physical world stimulates the construction of logical-mathematics knowledge, which in return supplies frameworks and structures that allow one to act on physical reality and to understand it". It seems therefore important to propose many connections between these two sciences when teaching. Yet, when examples related to physics are given in mathematics courses, excessive simplifications occur and the physics subject is used as a simple pretext to illustrate a mathematical concept. To create a *synergism of learning*, it is worth finding situations in which mathematics is felt necessary and effective for solving physics problems. Both the involved mathematics and physics have to be meaningful and unobvious and yet accessible to students and not too abstract or complicated. In this way, mathematics must not be seen as a calculation technique but rather as a corpus of ideas, concepts and methods, even qualitative, that allow to treat real problems rationally. Moreover, this can help students to connect the exact world of mathematical models and the approximations that physical reality demands, to understand that physics, more than an "exact science", is a science that knows how to rationally manage its inaccuracy.

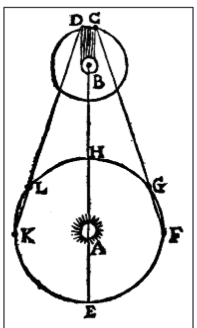

Fig. 1

2 History of science and physics education

The role of the history of science in teaching physics has been studied and debated at great length: it has been considered as one that gives cultural value to a learning not limited to technical instruments, favoring a more critical attitude and a conception of science as an evolving human activity. Research on common sense conceptions has renewed this debate from another standpoint because it has indicated that many of them are similar to ancient ideas or theories historically abandoned. It has been supposed that, confronted with such ideas and theories, students would have recognized their own conceptions, discussing and reviewing them, with conceptual change sequences similar to the historical ones. Such proposals have been criticized of their often too simplistic analogy between ancient theories and common thought. Nonetheless, this analogy exists in a limited form. Piaget & Garcia [4] emphasized it strongly, from the evolution of ideas point of view and, in some cases, in content as well. Moreover, the history of science could contribute to give sense to physics learning. In fact, the didactic transposition constraints to isolate and re-structure physics subjects in order to adapt them to the school context, thus hiding cultural and social references that defined the problems in answer for which scientific theories have been formed. This can produce a fragmentary, algorithmic more than conceptual, knowledge. Studying "case history" involving significant historical or philosophical aspects can contribute to giving back the atmosphere of debates, controversies and technical and economic backgrounds that constituted the context of science development.

3 A didactic unit on Römer experiment

Römer's experiment is interesting because it was part of the historical debate between the two conceptions of instantaneous propagation (Descartes, Kepler…) and finite speed of light (Galileo, Huygens…). Moreover, studying it more closely than textbooks normally do forces the use of unobvious mathematic tools adaptable to different educational levels.

The proposed didactic unit begins with a short presentation of the historical context and the basic ideas of Römer's astronomic method. A figure from the original article [5] (see figure 1) is used to illustrate how to measure the revolution period of Jupiter's satellite Io and how the changes in its apparent period can be related to the propagation of light from Io to Earth. The definitions of conjunction, quadrature, opposition, occultation and eclipse are provided. A scale drawing and an animation of Jupiter (J) and Earth (E) orbits around the Sun (S) are offered. The approximations of circular, uniform, coplanar orbits are declared Fig.1 and discussed.

Given the periods and diameters of the Earth's and Jupiter's orbits, and an average value of the Io period, students are asked to find the number of Io revolutions in one year, the angle covered by the straight line SJ during one year and the time between two successive oppositions J-S. These questions require the use of angles and time units and of simple equations, and the understanding of the relative motions of J and E.

Students are asked why it is not possible to see both the beginning and the end of the same Io eclipse and in which parts of the Earth's orbit the beginning or the end can be observed. This problem is linked to the relatively short distance between Io and J: students can be asked if this property also applies for other satellites. Reasoning involving similar triangles and proportions is needed here, along with a consciousness of spatial relationships and dimensions. The occultation and shadow cones of J are showed in a figure, with an animation (see figure 2).

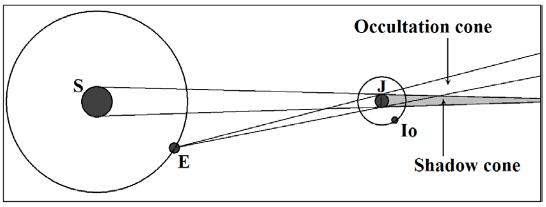

Fig. 2 Shadow and occultation cones

The changes of Io's apparent period *p* are examined, asking in which reciprocal positions of J, E, S it will be the longest, the shortest or equal to the true period p_o. This is a critical point because many students tend to link the longer period with the longer distance JE [6]. The relevant factor, when the Earth moves from E_1 to E_2, is not the distance JE nor the magnitude of the displacement E_1E_2 but the difference of distances □=JE_1-JE_2: it is $p=p_o+$□$/c$ (*c*=speed of light). So, the longest and shortest apparent periods are observed when □=±E_1E_2, that is at the two quadratures JES, which occur at equal distance JE; and the true period is observed when □=0, that is at the opposition JES, where the distance JE is minimal. A figure and an animation are provided showing some triangles JE_1E_2 in which E_1E_2 has the same magnitude, but the difference □ changes. Here, the basic properties of triangles are involved. At higher education levels, it is possible to propose the explicit calculation of the function *p(t)*, which requires basic trigonometry and calculus.

4 The measure of the speed of light

The changes of *p* being very little (less than 15 s), Römer used the cumulated times of many revolutions of Io. In the didactic unit, I propose to use the duration of 49 Io revolutions occurring more or less between an opposition and a quadrature of Jupiter with the Sun. Using a table of dates and times for some beginnings or endings of Io eclipses, students are asked to calculate the duration T_1 of 49 revolutions from quadrature to opposition and T_2 from opposition to the other quadrature. They can observe that $T_2>T_1$ and calculate the difference T_2-T_1. Then, students are asked to find the distances JE at the J-S quadrature and opposition and their difference □. The ratio $c=2$□$/(T_2-T_1)$ gives the speed of light. The values found for different sets of data will be different, so students have to calculate the arithmetic mean and compare it to the accepted value of *c*. To find distances JE, Pythagoras' theorem is sufficient, and for times T_i, the calculation of calendar dates and times.

Finally, a return to the historical context is proposed, with a discussion of the results found by Römer and Huygens and their meaning for physics and cosmology.

Conclusions

This short didactic unit can produce an effective synergism between mathematics and physics providing an interesting case study concerning a fundamental physics problem. The mathematic concepts and tools involved are numerous and can be dealt with at different levels. The first in-class experiments show a good student response and improvement in the understanding and interest on the related subjects. Some specific difficulties are observed, with a frequent correction of initial misunderstandings.

References

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