Complex Shadows

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Abstract

The concept of inferring reality from shadows is an important practical as well as theoretical one, that has implications for research in modern physics, for example in astronomy. The paper discusses a variation on the well known question , "Does there exist a three dimensional object that can give three different shadows which are, in particular, a circle, a square and an isosceles triangle?" An algorithmic model is presented for answering much more complex questions. This is suited for project based learning. I.e., the students can make nice diagrams and pose challenges to each other to solve their own inverse shadow problems. Also, famous examples of inverse shadow problems are classified and referenced.

Introduction

This paper describes two things about shadows, as they can be used in a physics class and that can be used to make drawings and sketches by the students.

1 Basic static shadow problems

Shadows are very frequent, everyday experiences both indoors and out. Since scientists want to study and explain all processes, and occurrences, shadows and their movements are of great interest to scientists, even today . A very obvious and natural question with regard to shadows is, *"What can we infer about an object from its shadow(s), assuming that the light source is given?"*
In a sense this involves working backwards with a model of the situation. Shadows are the effects of an object intervening into a light stream, and the question is whether, or to what extent, under what conditions is the inverse problem is solvable, by deriving a unique shape of the object. Such problems are known as inverse problems. They need a model to be answered. In this paper we consider only the simplest model:-
The binary model: There is either no shadow at all (transmits all the light) or a 100% shadow totally opaque, transmits zero light) . In the student drawings this model has only two values, say white and black. (More advanced versions of this paper would also consider an *X-ray type model*, wherein the amount of light transmitted is inversely proportional to the amount absorbed by the material inside the object being illuminated. In student drawings this model has appropriate shades of gray.)

2 Inverse shadow problems

In a direct problem, the situation consists of conditions and the stated causes. These are given and the problem is to derive their effects uses. It is enough to make a model, and then plug in the initial conditions, and solve the model. Moreover, the model is usually given already. The parameters needed by the model are usually numbers or vectors. Each step follows from the previous ones in a logical fashion. In an inverse problem, on the other hand, the effects are given, and only a part of the situation is stated. The problem is to derive the full situation that gives the given effect.

2.1 A very interesting key case

Consider the binary model of ray optics. Assume further, that light is direct sunlight, so that the rays are (essentially) parallel. If the object is given the shadow can be simply calculated by drawing the appropriate light rays. Now suppose that the three shadows shown in figure 1 are given. The problem is to find a three dimensional object that will give such shadows–if it actually exists.

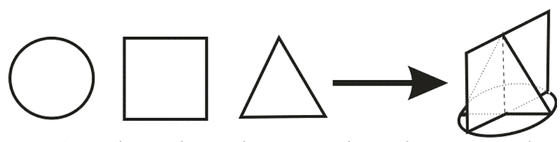

Figure 1 *Find an object that gives these three perpendicular shadows when illuminated by parallel rays.*

Ask the students to solve this. After a while give them hints of imagining a right circular cylinder. It is not easy for them to solve even then. After the end of the exercise, give them a new algorithmic method for determining the inverse. Explain it is an algorithm that works in many cases (Note: it has some limitations for pathologically non-convex shadows e.g., with holes in their interiors and/or deep indentations in their boundaries).

Algorithm. Step 1: Make cut-outs of the three given shadows, S*tep 2:* assemble the cut-outs so that they are mutually perpendicular as shown in figure 1.

2.2 Parallel cases for students' "research"/ play

The students are to (a) select and draw three convex forms, one for each of the three mutually perpendicular shadows, (b) cut them out, (c) assemble them to form an artifact of their inverse object, and (d) to test their artifact with light and shadows. Then, if they run into a problem that the resultant shadow is not quite the same as they started with, they are to go to steps (e) think why their situation almost works but not quite, and (f) revise their artifact and repeat steps (a) to (d) correctly. (Note: The limitation is that the extreme dimensions on any two of the perpendicular shadows imposes the same extreme positions on the third shadow.) They should figure this out by playing with very real objects such as rectangular boxes, where any two shadows determine the third completely. After a while they will get adept at drawing the three mutually perpendicular shadows consistently.

After they get the general idea, the students are then to challenge each other to solve inverse problems that they make.

More advanced students can think about cases where one or more of the given shadows has deep indentations or is laced with a grid of holes. Then the problem of assembling the three cut-outs becomes impossible without some modification of at least one cut-out so that it can "wind around the holes" without crossing them and changing the given original shadow with one or more "hair lines". Ask the students to also show why the above given algorithm is not a unique solution by adding extra planes that are small enough not to appear protrude through the boundaries of the three given shadows.

3 Classifying famous inverse problems

The Web has many very good articles not only on science lesson plans generally involving shadows but also on famous problems solved with the aid of shadow measurements and reasoning. A few website references together with a suggested classification are given next.

Classification of inverse problems with examples:

(a) find the shape of the object, given the shadow, and the light source (- Aristotle 384-322 BC hypothesized that the Earth was a sphere by the shape of the Earth's shadow as it swept over the Moon during a lunar eclipse.)

(b) find the distances between object and source given relative positions and shadows - Aristarchus of Samos 310-230 BC very roughly measured the size and distances of the Sun and Moon [1]. The transit of Venus across the surface of the Sun has a major histroical significance for the firast accurate measurement of this distance [2].

(c) find the viewing surface or "screen" on which the shadow falls, given the object, the shadow and the light source - Eratosthenes of Cyrene, 276-194 BC measured the circumference and tilt of the Earth [3], [4]

(d) Find out something about the shape of an object from shadows on its surface. - Galileo 1564-1642 calculated the height of mountains on the moon [5], (He also painted (oil on canvas) phases of the moon and made black and white drawings also [6], [7]. He also observed the full phase of Venus, supporting Copernicus' theory.)

(e) Find the speed of light from observations. Ole Roemer 1644- 1710 roughly measured the speed of light from observations regarding the periodic delays and advancements of the time when Io, one of the Galilean moons comes into Jupiter's shadow [8].

Extra Points

Can humans observe arbitrarily large accelerations on Earth? Yes we can. Imagine shadows of a flock of birds flying at an angle to a building. As the shadows cross the corner of the building they suddenly change velocity by an abrupt 90 degrees or pie/2 radians. Assuming that the corner in arbitrarily sharp, this involves an arbitrarily large deceleration in the direction the shadows were traveling, and at the same time an arbitrarily large acceleration in their new direction. This is quite an experience to observe.

Note: For a useful discussion ask students why the sharpness of the corner of the building is important in determining the acceleration of the shadows.

Until recently, astronomers could only infer the existence of extrasolar planets from the wobbling motion of their parent stars, indicating that the planet and the star are going around the barycenter around which the entire system resolves). Luckily star HD 20945, just 150 light-years away in the constellation Pegasus has a planet that orbits in a plane that is seen edge-on from Earth. Besides the wobble, during the planet's transit, the starlight observed on Earth is diminished by 1.7 percent. [9].

References

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