Entropy from the Beginning

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Abstract

When dealing with the concept of heat, the layman does not have particular problems: Heat is contained in hot bodies. The greater and the hotter the body, the more heat it contains. Heat goes or flows by itself from hot to cold, or from hot to less hot. Heat is produced within a flame, by mechanical friction or in the wire of a light bulb.

It is strange, that physics has trouble with the concept of heat. Simple statements about the every-day heat become incorrect, when interpreted in terms of the physical process quantity Q. They become correct, however, when the every-day heat concept is identified with the physical quantity "entropy" instead of Q. At the same time, one gets a very simple intuition about entropy, otherwise infamous for its abstractness.

A teaching sequence corresponding to the first five lessons of a thermodynamics course for beginners is presented, where entropy is introduced from the very beginning. In this short period of teaching time, not only the second and the third principle are introduced, but we also come to a physical description of basic processes of our everyday "thermal experience". Entropy appears as a quantity no more difficult than length, time or mass.

1 Introduction

"Entropy from the beginning". I have to explain what is meant by this title. It has nothing to do with the big bang. Of course, there was entropy since the beginning of all ages. But that is not what I mean. I simply want to say: When at school you begin teaching heat, then begin with entropy.

But isn't entropy a concept which is far too difficult to begin with? And isn't it true, that you can do a lot of good thermodynamics without this concept? The answer to both questions is "no".

In the first part of my talk we shall see why. In the second part, I shall sketch the beginning of a thermodynamics course for the High School. In the first part you are what you are, that is: the attendants of this conference, in the second you are pupils in a classroom.

2 An analogy

Physics is full of symmetries and analogies. Take as examples

- the analogy between electric and magnetic fields which reflects itself in the symmetry of Maxwell’ s equations,
  or
- the analogy between translational and rotational mechanics.

I will speak about yet another analogy, which comprises a substantial part of the whole of physics. Table 1 shows how it works. In this analogy, the extensive or "substance-like" quantities electric charge Q, momentum $\boldsymbol{p}$, entropy S and amount of substance n correspond to each other. The same holds true for the conjugated intensive quantities electric potential φ, velocity $\boldsymbol{v}$, absolute temperature $T$and chemical potential μ. To each of the extensive quantities a flow or current exists: the electric current I, the momentum current or force $\boldsymbol{F}$, the entropy current I_S and the substance current I_n.

Many of the relationships that exist between the quantities of one subfield of science (one line in the table) have a counterpart in another subfield. An example is shown in the last column of Table 1. Each of the equations in this column represents a description of an energy transport. The first equation (second line) corresponds to the so-called electric energy. (The letter U stands for an electric potential difference.)

Table 1.

subfield of science	extensive quantity	conjugate intensive quantity	current	energy flow
electricity	electric charge Q	electric potential φ	electric current I	$P = U \cdot I$
mechanics	momentum $\boldsymbol{p}$	velocity $\boldsymbol{v}$	force F	$P = \boldsymbol{v} \cdot \boldsymbol{F}$
thermodynamics	entropy S	absolute temperature T	entropy current I_s	$P = T \cdot I_s$
chemistry	amount of substance n	chemical potential μ	substance current I_n	$P = \mu \cdot I_n$

If the pertinent relation is that of the third line, then the energy exchange is called “work”. The equation of the forth line describes a transport in the form of heat and that of the last line corresponds to chemical energy.

In the *Karlsruhe Physics Course* [1, 2] we take extensively profit of this analogy. Here, I will draw only one conclusion: If we take this analogy seriously, then we can say, that each extensive quantity should be the protagonist of the corresponding subfield of physics:

Electricity is that part of physics which deals with electric charge and electric currents. Mechanics is that part of physics which deals with momentum and momentum currents. Thermal physics is that part of physics which deals with entropy and entropy currents.

And we can also conclude that:

Thermodynamics without entropy and without entropy currents is like mechanics without momentum and without force or like electricity without electric charge and without electric current.

For our teaching we deduce:

In electricity: electric charge from the beginning. In mechanics: momentum from the beginning. In thermal physics: entropy from the beginning.

You remember: That is the title of my talk.

3 The traditional concept of heat

Thermodynamics or thermal physics as it is taught normally is one of the awkward parts of physics, a kind of blemish. Why?

Thermal physics has to do with temperature and heat. The layman has a sound intuition for both: Temperature measures what we qualitatively describe by “hot” and “cold”. Heat is for the layperson what is in a hot-water bottle: If there is much water in the bottle, and if the water is hot, then, there is much heat within the bottle. The layman also knows, that, if we wait for a long while, the heat comes out of the bottle, it goes to the colder environment. He or she knows, that heat can be produced in several ways: In a flame, in an electric wire or by friction.

Now the physicist: As far as temperature is concerned, he or she has the same concept as the layman. However, the physicist’ s understanding of the concept of heat is different. You remember, that when you learnt about heat in the physics lessons at school or in the university lecture, you have been told some strange things, for instance:

When you deliver heat to a body, the body gets warmer. Okay, that’ s not strange. However, you are also told that it is not correct, or not allowed to say, that now the body contains more heat. We add heat to the body, but the body does not contain more heat. How that? W ell, you are told, that the internal energy of the body increased! Ah, then, heat is internal energy, you might conclude. No, you are told, heat is not internal energy. You add heat, and thereby the internal energy increases.

I know that many students are convinced that actually the heat of the body increases. They believe that simply they are not allowed to say so. I hope that *you* all know, that there is no heat, which increases, and that this is not only a way of speaking.

Let us take another example, which is even more intricate. Water is boiling. I add, say 15 kJ of heat. Everybody who is not “professionally deformed”, will expect, that the evaporated water has 15 kJ more heat than when it was in the liquid phase. But using the physicist’ s concept of heat, that is not true, as you probably know. Physics tells us, that not the *heat* of the vapor has increased by 15 kJ, but its *enthalpy*.

W e physicists got used to this unpleasantness, and I am afraid that most of us believe, that we have to do with an intrinsic difficulty of thermal physics. We are told and we believe, that thermal physics is so complicated. And possibly, many of us are convinced that the layman is not right.

W e will see later, that the layman *is* right, and we shall see, that the afore mentioned verbal acrobatics is not a necessary consequence of the laws of physics. We also will see, that thermal physics is not intrinsically difficult. Actually, the complications arise from an awkward presentation of thermal physics. They are due to the fact that traditional thermodynamics does not take profit of the simple properties and the simple behavior of entropy.

In order to see what is wrong, we next have to have a short look on the history of the concept of heat.

4 Some remarks about the history of the concept of heat

A physical quantity “heat” entered in physics at the end of the 18th century. It was introduced by the Scottish Chemist Joseph Black (1728-1799). Black was the first who clearly distinguished the intensive quantity “temperature” and the extensive quantity “heat”. He also introduced the concepts “specific heat” and “latent heat”. As a matter of course, his heat was a state variable.

This heat concept was a sound construction according to the standards of physics, and it was completely accepted at this time. It goes without saying that it was the same concept which was used by Sadi Carnot (17961832), who, in his famous“ Réflexions sur la puissance motrice du feu” studied how much work can be realized when heat goes in a thermal machine from a higher to a lower temperature. This was in 1824. The French term for heat used by him was alternately “chaleur” or “calorique”.

Then, about 25 years later a misfortune happened to the concept of heat. It was the fact that Joule (1818-1889), Mayer (1814-1878), Helmholtz (1821-1894) and Lord Kelvin (1824-1907) introduced energy. Of course, the misfortune was not the energy. Rather it was the fact that the scientific community was too enthusiastic about the new quantity. They also introduced a new concept for the description of thermal energy transports. This concept was analogue to the concept of work, which existed before, and which served, as you all know, to the description of mechanical energy transports. Both these concepts – heat and work – are what later was called process quantities, as opposed to all the other physical quantities, which sometimes are called state variables. The misfortune was, that the new process quantity was called "heat". One might ask, why they chose a name, which was already used for another quantity. The answer is simple: They believed, that the old heat of Black was the same quantity as the new process variable. They believed, that only Black and Carnot had not realized, that their heat was an energy form, i. e. a quantity of the same type as mechanical work. Of course, with this interpretation, some of Black's and Carnot's conclusions were in contradiction to those of the energeticists. From this, they concluded, that Black and Carnot were mistaken in some points. Nobody noticed, that they simply were in presence of two different physical quantities. Seen from a modern point of view, Black' s quantity is exactly, what nowadays is called entropy [3]. At that time however, it was not noticed, that they had to do with two different quantities.

Recapitulating: The name "heat" was attributed to the newly invented energy form, and it was imputed to Black and Carnot, that they had tried to introduce this energy form, but that they had failed in some details.

The consequences of this misfortune were catastrophic for thermodynamics [4]. Not only, that the most important heat measure disappeared from physics, but even worse, its name had been stolen. The name "heat", "chaleur" or "Wärme" was now given to a form of energy. And from now on, it was forbidden to say, that a body or a system *contains* heat. The quantity, which was now carrying the name heat, was hard to handle. Simultaneously, Black's simple and important "amount of heat" had disappeared from physics.

Henceforth, there was no way to express the amount of heat contained by a system. It is evident, however, that such a measure is indispensable for physics and chemistry. As a consequence, several more or less unsatisfying Ersatz or surrogates appeared, the best known being the enthalpy.

But this changed nothing with the fact, that the old quantity of Black and Carnot was still missing. This was evident from the thermodynamic formalism. So, from our actual point of view, it was not a surprise at all, that the old quantity found its way back into physics. However it appeared as something new and it was given a new name: It was the entropy, introduced by Clausius in 1865. At that moment, nobody was aware that this quantity was not new at all but only a resurrection of Black' s heat. And nobody noticed that entropy was no other thing than the heat concept of the average citizen. This strange state of thermodynamics persisted for almost half a century. Only in 191 1, the British thermodynamist H. L. Callendar (18631930) became aware of it and he revealed his discovery in a beautiful article [5] in the *Proceedings of the Physical Society of London*. I cite only a few sentences from this article: "Finally, in 1865 when its importance [the importance of caloric] was more fully recognized, Clausius gave it the name of 'entropy', and defined it as the integral of *dQ/T*. Such a definition appeals to the mathematician only. In justice to Carnot, it should be called caloric, and defined directly by his equation [...], which any schoolboy could understand. Even the mathematician would gain by thinking of a caloric as a fluid, like electricity, capable of being generated by friction or other irreversible processes."

Unfortunately, this discovery came too late. Apparently, a concept which is firmly established over 50 years, has no chance to be changed. Accordingly, Callendars work remained almost unnoticed, and the physics community went on fighting with an inappropriate heat concept and a seemingly incomprehensible entropy.

So it is no wonder that sixty years after Callendars publication, the inconsistency was anew discovered. In 1972 appeared a textbook about thermodynamics, in German language, with the title: "The entropy as heat" [6]. The autor was Georg Job, who is here among us, and who at that time was not aware of Callendars work. Now of course he is. His book is actually being translated into English and into Spanish. More recently, Fuchs seized the idea in his book "The Dynamics of Heat" [7].

5 Entropy as heat

I now come back to the question: "Isn't entropy too difficult to be taught at school?"

I said already "no", and we can now see why this is true.

Entropy is that quantity, which, in former times was called heat. This name was absolutely justified, because the correspondence between the everyday concept of heat and the entropy of physics is so perfect, that when taking profit of it, teaching entropy becomes easier, than most other physical quantities. According to our experience, I can affirm, that:

Entropy is one of the easiest quantities of physics, comparable to length, time and mass.

Also according to our experience, I can assert that the layman, and that means also our pupils, have a solid knowledge of the second principle before they have their first physics lesson.

6 The first five lessons

In the following, I will sketch the first five lessons of a course of thermal physics. These are part of the *Karlsruhe Physics Course* [1]. The course was designed for 14 or 15-year old pupils. But in the last 20 years it was tested with pupils of many other age groups: from 13 year olds to 18 year olds.

The five lessons, which I shall sketch, are compressed by a factor of 10 approximately. I address to you as if you were my pupils. I try to sketch the words of the teacher as well as those of the pupils, such as they are typical, and as they are known to me from teaching this unit many, many times. So, let us begin.

A bell rings.

Teacher: Today we begin with thermal physics: We are concerned with the being hot and cold of a body. You all know that physicists describe the world quantitatively. And for that purpose, they use physical quantities. You all know such a quantity, which allows for a description of being hotter or colder.

Pupil: Temperature.

Teacher: Yes, that's right.

We introduce the symbol and the measuring unit, for the moment Centigrades.

Teacher: With one single quantity we cannot do physics. Physics asks for relations between various quantities. Therefore, we need yet another quantity with which describe the hot and the cold. And this second quantity, I'm sure you also know it. It tells us, how much heat a body contains, in other words: the amount of heat. A hotwater bottle for instance contains heat, provided that the water in the bottle is hot. Well, that's our second quantity. Now, physicists usually give a particular name to a quantity, often a name with Greek origin, in order to avoid confusion with some other concept. In physics, the amount of heat which we just mentioned, is called entropy.

The abbreviation or symbol is *S,* and the name of the unit is Carnot, abbreviated Ct. To get an idea about how much is one Carnot: 1 cm^3 of water at standard temperature contains 4 Ct approximately.

Let us see if you have understood. I'll ask some questions, in each question there is the word entropy. Every time I say entropy, you mentally translate into "amount of heat".

So, look at this: This is cold water, about 10 degrees, and that is hot water, 70 degrees. It's the same amount. In which beaker there is more entropy?

Pupil: In this one, the one with the hot water.

Teacher: Okay. Now, let's do something else. Again two beakers, but this time, the temperature is the same in both. However, there is more water in this one than in the other one. In which one there is more entropy?

Pupil: In the one with more water.

Teacher: We thus have the rule:

The higher the temperature of an object, the more entropy is contained in it.

The greater the mass of an object, the more entropy is contained in it.

Now wait. Imagine that in this beaker there are 12 Ct. I pour one third of this water into the other beaker. How much entropy is now in this beaker?

Pupil: 4 Ct.

Teacher: Right. And where are the remaining 8 Ct?

Pupil: In the other one.

The bell rings.

This was the first lesson. Probably you have understood, what the preceding exercises have been good for: It is to make the pupils understand the extensive character of the entropy and to distinguish it from temperature.

Back to our virtual classroom and the second lesson.

Teacher: Here is a beaker with hot water, which I dip into cold water. What will happen?

Pupil: The temperature of the water in the beaker will go down, and the one of the outer container will go up.

Teacher: Can you explain me why? Remember, that when I ask an explanation I mean that you tell me what happens with the entropy.

Pupil: The entropy goes from the inner to the outer container.

Teacher: Could you formulate a rule? Is it right, that the entropy always goes from inside to outside?

Pupil: The entropy goes from hot to cold.

Teacher: Okay, we thus have the rule:

Entropy flows by itself from places of higher temperature to places of lower temperature.

As this happens, the warmer body gets colder, and the colder gets warmer. In other words: The temperature difference decreases. When the temperatures have leveled completely, the entropy flow will stop. The state, which is now attained, is called *thermal equilibrium.*

We can also express this in the following way:

A temperature difference is a driving force for an entropy flow.

The bell rings.

This was the second lesson. We come to the third one.

Teacher: We have seen, that entropy goes on its own from hot to cold. It goes alone downhill the temperature slope, on its own initiative. However, it may happen and it often happens that we want the entropy to go uphill, that is from cold to hot. How can we do? ... You don't look like having an idea. However, that's not a far-fetched problem. You know for instance, that the air goes spontaneously form a place of higher pressure to a place of lower pressure. It comes out of a punched tire, it does not go in. What do we do if we want the air to go into the tire?

Pupil: We use a pump.

Teacher: Yes, we force the air to go inside, against its natural tendency. If we want the entropy to go from cold

to hot, against its natural tendency, we also have to force it. And even if you don' t know how to do it, you can tell me how we should call the device, which helps us to do so.
Pupil: An entropy pump.

I now tell them, that they all have an entropy pump in their home, and after some time they get aware about what I'm talking: about the refrigerator.
Teacher: By the way, the technical term of an entropy pump is *heat pump.* The heat pump of the fridge conveys the entropy from the inside to the outside.

We study an authentic fridge in the classroom, and we discuss details. I've to skip that here.

The bell rings.

I come to the forth lesson. Teacher: Here in my hands, I have a brick. And I ask: How much entropy contains the brick? How much entropy can I extract from it?

It's early in the morning, and my pupils seem to be a little dozy.

I ask them if they have noticed that I have asked two questions.
Pupil: Why two? It was one.
Teacher: No, it was two: 1. How much entropy is in it and 2. how much can I get out.
Pupil: But that's the same thing, isn't it? When I have a bottle of schnapps, and there is one liter in it, I can get one liter out.
Teacher: That's okay for the schnapps, but I know situations, where you can get out more than there is inside.

(After a moment of reflection)
Pupil: O yes, my bank account. I had sixty euro on the account and the bank gave me a hundred.
Teacher: And then, what happens with your account?
Pupil: Well, the account is now overdrawn.
Teacher: And that means, you now have a debt of 40 euros, or your possession is minus 40 euros. There are other examples for such a situation that are more related to physics.

With some help, the pupils remember electric charge, and momentum. From a positively charged body we can withdraw more charge than what is on it, and as a consequence the body will be charged negatively.
Teacher: Now, how about entropy? Well, we don't know, so we have to make an experiment. We have to take entropy out of the brick, more and more, and see what happens. Unfortunately, we can't do this in the classroom, since the heat pump we would need is far too expensive. But as a matter of course, scientists have tried it with all means. In the last 150 years, they have constructed heat pumps which got better and better, and they finally made an interesting observation: Whatever the effort you make, you don't get below a temperature of minus 273 °C. This is the lowest temperature, which can be attained. Can you give me the reason for that?
Pupil: The temperature doesn't decrease anymore upon pumping because there is nothing left to pump. There is no entropy left in the brick.
Teacher: And that would be true for any other body at 273 °C.

The lowest temperature that an object can have is –273.15 °C. **The object contains no entropy at this temperature.**

We thus have also the answer to our second question: There is no negative entropy. We cannot bring out more entropy than there is inside.

We now define a new temperature scale: We take as a zero point this lowest temperature. And when measured on this scale we call it the absolute temperature.

On the absolute scale, zero is at –273.15°C. The unit of the absolute temperature is the Kelvin.

The bell rings.

This was the forth lesson. You may have noticed that we learned the third law of thermodynamics. We come to the fifth and last lesson, which I will present.
Teacher: We want to heat a room. We know what that means: We have to augment the entropy content of the room. How can we do?
Pupil: Turn on the heating.
Teacher: But what's going on when the heating is working?
Pupil: Somewhere in the basement, there is a boiler and in the boiler there is a flame. The entropy is coming from this flame.
Teacher: And how did it get into the flame? *Pupil:* It did not get there; it originates in the flame. It is produced within the flame.
Teacher:

Entropy can be created by chemical reactions (for example burning).

W e could heat the room yet in another way, with an electric heater. We are looking for the origin in this case and find:

Entropy can be created in a wire with an electric current.

Finally, entropy can be created with a third method:

Entropy can be created by mechanical friction.

The following is particularly interesting. I say that we now don't want to heat something, but to cool.
Teacher: How can we do? For instance: Your tea is too hot; it contains too much entropy; you want it to be cooler.
Pupil: Just wait.
Teacher: Right. And what happens then with the entropy?
Pupil: It comes out; it goes into the room, into the environment.
Teacher: But the room doesn't get warmer!
Pupil: Yes, it does.
Teacher: But I don't notice it.

Pupil: That's normal, because the room is so big and the teacup is so small.
Teacher: But couldn't we cool the tea yet in another way?
Pupil: Yes, of course. We can put it into the fridge.
Teacher: Ah yes. And where does the entropy go in this case?
Pupil: It comes out behind, through the black pipes.

I try to lead the pupils on a wrong track, but I don't succeed. So I say it more directly:

Teacher: You told me, that entropy can be produced. Entropy appears, which was not taken away from somewhere else. Couldn't we get rid of the entropy in a similar way: annihilate it definitely.
Pupil: No.
Teacher: Why not?
Pupil: Because that's the way it is.

Apparently, the pupils have a clear idea about the second law. They have it from their everyday experience. I tell them that we have just discovered a law that is one of the most fundamental laws of physics:

Entropy can be created but not destroyed.

The pupils don't find this result very exciting. I tell them, that the scientists found this discovery absolutely egregious, and they tried everything to show that it is not true. Only now, the pupils begin to cogitate.

Pupil: Oh yes, if this is true, then the entropy of the Earth must increase more and more.

We discuss the consequences for the Universe.

Teacher: This law is also interesting, because it tells us that the time cannot run backwards. If in a process entropy is created, then this process cannot run backwards, since that would mean that entropy is annihilated, and that is forbidden according to our law. We thus have.

Processes in which entropy is produced are irreversible.

I have to stop here. You have noticed, that in these five lessons, not only the second law but also the third law has been introduced. In the next lessons we discuss entropy transport by conduction and convection. Then, we introduce the relation between entropy and energy currents. The pupils are now able to calculate the energy and entropy balance of heat pumps, thermal power plants and heating systems. They also are able to calculate efficiencies. Later we introduce the entropy capacity (equivalent to heat capacity), we discuss the thermodynamics of gases and phase transitions and some thermodynamics of electromagnetic radiation. The whole corresponds to about 30 "hours" (of 45 minutes) of teaching time.

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